Real Analysis

Math 310 — MWF 10:30-11:25am

Fall 1996 — 127 Fowler

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#23 Monday (11/11/96)

SUMMARY Review for Exam 2 / Continuity continues

READING §17 (entirety) and §19 (page 100-102)

Warm-Up

We shall go over the proof of how to show that $f(x) = 1/x^2$ is continuous at $x = x_0$

Example

Show that $f(x) = \sqrt{x}$ is continuous at $x = x_0$ (HINT: We shall have to split this up into the case where $x_0 = 0$ and $x_0 \neq 0$)

Exercise

Show that if $f(x) = x^2 \sin(1/x)$ for $x \neq 0$ and $f(0) = 0$, this function is continuous at $x = 0$.

Exercise
Show that if $f(x) = (1/x)\sin(1/x^2)$ for $x \neq 0$ and $f(0) = 0$ then this function is **discontinuous** at $x = 0$.

Definition A function $f(x)$ is *DISCONTINUOUS* at a point x_0 if and only if ...

Exam 2 Topics

- 10.6 Definition of lim sup and lim inf
- 10.7,10.8,10.9,10.10,10.11 Cauchy sequences (definition and properties)
- 11.2, 11.3, 11.4 Subsequences (definition, selection function)
- 11.5,11.6,11.7,11.8 Properties of subsequences, subsequential limits, Bolzano-Weierstrass Theorem, etc
- 12.2 $\liminf |\frac{s_{n+1}}{s_n}| < \liminf |s_n|^{1/n} < \limsup |s_n|^{1/n} < \limsup |\frac{s_{n+1}}{s_n}|$
- §14 Series Notation, partial sums
- definition of convergence/divergence, absolute convergence
- Geometric series, p-th test, Cauchy criterion
- Convergence Tests: Comparison, Ratio, Root, Alternating Series, Integral
- §17 both definitions of continuity ($\epsilon - \delta$ and limit definition
- Ability to prove continuity/discontinuity at a point
- Definition of Uniform Continuity, proof of uniform continuity on a set

Sample Questions

1. CONTINUITY/UNIFORM CONTINUITY (a) Use the ϵ-δ definition of uniform continuity to prove that $f(x) = 3x^2 + 1$ is uniformly continuous on the interval $[-2, 2]$.
(b) Prove that $f(x)$ given below is discontinuous at $x = 0$:

$$f(x) = \begin{cases} 1, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

2. SUBSEQUENCES
For the sequence $(a_n) = (-2)^n$:
(i) Give an example of a monotonic subsequence of (a_n).
(ii) Give the set, S, of subsequential limits of (a_n).
3. SERIES
Do the follow ing series converge or diverge? Fully support your answers with appropriate tests – you must tell me which test you are using, and you must correctly show me the use of that test.
(a) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ (b) $\sum_{n=1}^{\infty} \frac{-4}{2^n}$

This is not intended to be an exhaustive list of the topics or questions to be asked on exam 2, but should give you some sense of what to expect on the exam on Friday November 15.