Real Analysis

Math 310 MWF 10:30-11:25am
Fall 1996 127 Fowler

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#22 Friday (11/01/96)

SUMMARY Continuity and Uniform Continuity

READING pages 87-93 §17, §19

I. Write the ϵ-δ definition of **continuity of** $f(x)$ **at** x_0.

II. What quantities does your choice of δ depend on?

III. Let's work specifically with $f(x) = 1/x^2$.

A. GEOMETRIC OBSERVATIONS

1. Where is this function continuous?

2. What are $f(0.1)$, $f(0.2)$ and $f(0.25)$? Mark these values on the y-axis of the accompanying graph.

3. From each of these values ($f(0.1)$, $f(0.2)$ and $f(0.25)$) on the y-axis, draw a horizontal line to the points $(x, f(x))$ on the graph, then vertical lines down to the x values (0.1, 0.2, 0.25).
(I have done the lines for $f(0.1)$ for you.)

4. Rewrite the definition of continuity using the function $f(x) = 1/x^2$ and the point $x_0 = 0.2$.

5. [Use a different color pen from 3.] Let $\epsilon = 5$. Draw this ϵ-neighborhood around $f(0.2)$ on the y-axis of the graph (i.e. draw a little interval from 5 units above $f(0.2)$ to 5 units below). Now follow these y-values with horizontal lines over to the graph of $1/x^2$ and then down to points near $x = 0.2$ on the x-axis to see what corresponding δ-neighborhood you get. Using the picture, approximate what δ you must choose.

6. Find the exact δ we need to choose for $\epsilon = 5$ by using a calculator and the function $1/x^2$.

7. Repeat 5 & 6 with $x_0 = 0.1$.

8. Repeat 5 & 6 with $x_0 = 0.25$.

9. After this, do you change and/or believe your answer to II? (Explain the difference between your answers to 5/6, 7, and 8.)

B. ANALYTIC PROOF

1. Using the ϵ-δ definition of continuity, prove that $f(x) = 1/x^2$ is continuous on the interval $(0, \infty)$. Basically what I want you to do is figure out what your choice of δ should be for an arbitrary $\epsilon > 0$. You'll see the dependence on both x_0 and ϵ! (Hint: When you get a "formula" for $f(x) - f(x_0)$, assume that $|x - x_0| < \frac{x_0}{2}$ and show that this implies $|x + x_0| < \frac{5x_0}{2}$.) Your work will be similar to Example 1(b) in the text.

2. FORMAL PROOF

Compare the choice of δ you get from this proof (say for $\epsilon = 5$, $x_0 = 0.2$) with what you got in 5 on the previous page.

IV. Let us say that f is UNIFORMLY CONTINUOUS on a set S when the choice of your δ does not depend on the points in S (the x_0). Try to write a definition for UNIFORM CONTINUITY on S.

Compare with **Definition 19.1** in Ross. If it is different than your definition, then understand why...and make the appropriate changes above.

Why does it not make sense to talk about uniform continuity at a point?

Why must you talk about uniform continuity on a set?

Homework Assignment (Due Wednesday November 4)

§17, page 56: **17.1, 17.3(a),(c),(g), 17.4, 17.9(b),(c), 17.10(b)**