Real Analysis

Math 310 MWF 10:30-11:25am
Fall 1996 127 Fowler

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#21 Wednesday (10/30/96)

SUMMARY Continuity

READING pages 87-93 §17

Warm-Up
(5-10 minutes) Draw a picture of a **continuous** function and a picture of a **discontinuous** function

Can you think of a way to link the idea of continuity with the idea of sequences?

Definition

17.1 Let f be real-valued function whose domain $\text{dom}(f) \subseteq \mathbb{R}$. The function f is said to be **continuous at** x_0, where $x_0 \in \text{dom}(f)$, if for EVERY sequence (x_n) in $\text{dom}(f)$ where $\lim x_n = x_0$ we have $\lim f(x_n) = f(x_0)$

The function f is said to be **continuous** if it is continuous at EVERY point in $\text{dom}(f)$.

Definition

17.2 Let f be real-valued function whose domain is a subset of $\mathbb{R}$. Then f is **continuous at** x_0, where $x_0 \in \text{dom}(f)$, *if and only if*

$$\forall \epsilon > 0 \quad \exists \delta > 0 \quad \ni \quad x \in \text{dom}(f) \text{ and } |x - x_0| < \delta \Rightarrow |f(x) - f(x_0)|$$

Example

Show that $f(x) = x^2 + 1$ is continuous at $x = 2$

Consider the function $f(x) = x^2 \sin(1/x)$ for $x \neq 0$ and $f(0) = 0$
Use `Derive` to graph this function. Does it LOOK continuous at $x = 0$?

Exercise

Show that $f(x)$ is continuous at $x = 0$.

Exercise

Show that if $f(x) = (1/x)\sin(1/x^2)$ for $x \neq 0$ and $f(0) = 0$ then this function is **discontinuous** at $x = 0$.
Use `Derive` to graph this function. Does it LOOK continuous at $x = 0$?

Homework Assignment (Due Monday November 4)

§17, page 56: **17.1, 17.3(a),(c),(g), 17.4, 17.9(b),(c), 17.10(b)**