Real Analysis

Math 310 — MWF 10:30-11:25am
Fall 1996 — 127 Fowler

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#20 Monday (10/28/96)

SUMMARY Finishing Infinite Series, Continuity of Functions

READING §14-15 (pages 70-79)

Cauchy Criterion
The series $\sum a_n$ satisfies the Cauchy Criterion if its sequence of partial sums (s_n) is a Cauchy sequence:

$$\forall \epsilon > 0 \exists N \ni m, n > N \Rightarrow |s_n - s_m| < \epsilon$$

If we assume that $n > m$ and that we can use $m-1$ in place of m then this becomes:

$$\forall \epsilon > 0 \exists N \ni m-1, n > N \Rightarrow |s_n - s_{m-1}| < \epsilon$$

which is equivalent to:

$$\forall \epsilon > 0 \exists N \ni m-1, n > N \Rightarrow |\sum_{k=m}^{n} s_k| < \epsilon$$

Theorem
(14.4 $\sum a_n$ converges **if and only if** it satisfies the Cauchy Criterion
PROOF:

Corollary
If a series $\sum a_n$ converges, then $\lim a_n = 0$
PROOF:

Conjecture If $\lim a_n = 0$, then $\sum a_n$ converges.
TRUE or FALSE?
PROOF

Theorem
$\lim a_n \neq 0$, then $\sum a_n$ diverges.
PROOF:

Let's prove that the COMPARISON TEST works for convergence.
If $\sum a_n$ (where $a_n > 0$) converges, and $|b_n| \leq a_n \quad \forall n$, then $\sum b_n$ ALSO converges

Theorem
Let's prove that the ROOT TEST works for convergence.
Let $\sum a_n$ be a series and then if $\alpha = \limsup |a_n|^{1/n}$ and $\alpha < 1$, then $\sum a_n$ converges.

Theorem
Let's show how the RATIO TEST proves convergence.

Theorem
15.1 $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges if and only if $p > 1$
Let's use the Integral Test to prove this.

Homework Assignment (Due Friday November 1)
§14, page 75: **14.1, 14.3, 14.4(b), 14.8, 14.13** §**15, page 79: 15.1, 15.3, 15.6**