Real Analysis

Math 310 — MWF 10:30-11:25am
Fall 1996 — 127 Fowler

Ron Buckmire ~ 402 Fowler ~ 259-2536 ~ ron@oxy.edu ~ Mon 1:30-3:30pm Wed & Thu 1:30-2:30pm

#19 Friday (10/25/96)

SUMMARY Infinite Series

READING §14-15 (pages 70-79)

Warm-Up

(5-10 minutes) Consider $\sum_{n=1}^{\infty} \frac{n}{2^n}$ Use the following tests to determine convergence

- Comparison Test
- Ratio Test
- Root Test

Integral Test
Compare your series to some integral

(i) Compare your series to some improper integral

(ii) If you think your series DIVERGES, make it $\geq$ a DIVERGENT integral

(iii) If you think your series CONVERGES, make it $\leq$ a CONVERGENT integral

Example
Let's use the **Integral Test** to show that
(a) $\sum_{k=0}^{\infty} \frac{1}{k}$ diverges

(b) $\sum_{k=0}^{\infty} \frac{1}{k^2}$ converges

When should we use the integral test on a series?
Alternating Series Test
15.3 If $a_1 \geq a_2 \geq \ldots \geq a_n \geq \ldots \geq 0$ and $\lim a_n = 0$ then
the alternating series $\sum (-1)^n a_n$ converges
Example
Consider $\sum \frac{(-1)^n}{n}$
(a) Does this series CONVERGE?

(b) Does this series CONVERGE ABSOLUTELY?

Cauchy Criterion
The series $\sum a_n$ satisfies the Cauchy Criterion if its sequence of partial sums (s_n) is a Cauchy sequence:

$$\forall \epsilon > 0 \exists N \ni m, n > N \Rightarrow |s_n - s_m| < \epsilon$$

If we assume that $n > m$ and that we can use $m - 1$ in place of m then this becomes:

$$\forall \epsilon > 0 \exists N \ni m - 1, n > N \Rightarrow |s_n - s_{m-1}| < \epsilon$$

which is equivalent to:

$$\forall \epsilon > 0 \exists N \ni m - 1, n > N \Rightarrow |\sum_{k=m}^{n} s_k| < \epsilon$$

Theorem
(14.4 $\sum a_n$ converges **if and only if** it satisfies the Cauchy Criterion
PROOF:

Corollary
If a series $\sum a_n$ converges, then $\lim a_n = 0$
PROOF:

Conjecture If $\lim a_n = 0$, then $\sum a_n$ converges.
TRUE or FALSE?
PROOF

Theorem
$\lim a_n \neq 0$, then $\sum a_n$ diverges.
PROOF:

Let's prove that the COMPARISON TEST works for convergence.
If $\sum a_n$ (where $a_n > 0$) converges, and $|b_n| \leq a_n \quad \forall n$, then $\sum b_n$ ALSO converges

Homework Assignment (Due Friday November 1)
§14, page 75: **14.1, 14.3, 14.4(b), 14.8, 14.13 §15, page 79: 15.1, 15.3, 15.6**