Real Analysis

Math 310 — MWF 10:30-11:25am

Fall 1996 — 127 Fowler

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#18 Wednesday (10/23/96)

SUMMARY Infinite Series

READING §14 (pages 68-75)

Warm-Up
(5-10 minutes) Recall all that you know about infinite series. Write down an example of one. What notation are you using? How do you determine convergence? Etc..

Definition
Infinite series

$$s_n = \sum_{k=m}^{n} a_k =$$

s_n is called a ____________________

14.2 An infinite series is said to **converge** to a number S when

$$\sum_{n=m}^{\infty} a_n =$$

A series that does not converge is said to __________

If the terms a_n of an infinite series are nonnegative (≥ 0) then we know that (s_n) is a ____________________ sequence.

The series $\sum a_n$ is said to *converge absolutely* if $\sum |a_n|$ converges

The series $\sum(-1)^n a_n$ is said to be an *alternating* series if $a_n \geq 0$ (or $a_n \leq 0$) for all $b \in \mathbb{N}$

CONVERGENCE OF INFINITE SERIES

Geometric Series

Try and recall formulas about the convergence of geometric series.

$\sum_{n=0}^{\infty} ar^n =$

If $|r| < 1$ then $\sum_{n=0}^{\infty} ar^n =$

p-Series

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ CONVERGES **if and only if** $p > 1$

(14.6) Comparison Test

Let $\sum a_n$ be a series with $a_n \geq 0 \forall n$

(i) If $\sum a_n$ **converges** and $|b_n| \leq a_n$ for all n, then $\sum b_n$ converges

(ii) If $\sum a_n = +\infty$ **and** $b_n \geq a_n$ for all n, then $\sum b_n = +\infty$

(14.8) Ratio Test

A series $\sum a_n$ of nonzero terms

(i) **converges absolutely** if $\limsup |a_{n+1}/a_n| < 1$

(ii) **diverges** if $\limsup |a_{n+1}/a_n| > 1$

(iii) Otherwise if $\liminf |a_{n+1}/a_n| \leq 1 \leq \limsup |a_{n+1}/a_n|$ and the test gives no information

(14.9) Root Test

Let $\sum a_n$ be a series and let $\alpha = \limsup |a_n|^{1/n}$. The series $\sum a_n$

(i) **converges absolutely** if $\alpha < 1$

(ii) **diverges** if $\alpha > 1$

(iii) Otherwise if $\alpha = 1$ and the test gives no information

Homework Assignment (Due Friday October 25)

§11, page 56: **11.4, 11.5** §12, page 60: **12.3, 12.14(a)**