Real Analysis

Math 310 — MWF 10:30-11:25am
Fall 1996 — 127 Fowler

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#17 Monday (10/21/96)

SUMMARY Subsequences and $\lim inf$ and $\lim sup$

READING pages 54-60

Warm-Up
(5-10 minutes) For each of the sequences find the set of subsequential limits.
(a) Let (r_n) be a list of all the rational numbers.

(b) Let $(b_n) = n[1 + (-1)^n]$ for $n \in \mathbb{N}$

Theorem
11.7 Let (s_n) be any sequence in $\mathbb{R}$ and S denote the set of subsequential limits of (s_n)

(i) S is nonempty

(ii) $\sup S = \limsup s_n$ and $\inf S = \liminf s_n$

(iii) $\lim s_n$ exists **if and only if** S has exactly one element, namely $\lim s_n$

DISCUSS why each of these three statements "makes sense" to you...

Theorem
11.8 Let S denote the set of subsequential limits of a sequence (s_n). Suppose (t_n) is a sequence in $S \cap \mathbb{R}$ and that $t = \lim t_n$. Then t belong to S. Explain this theorem in your own words. What does it mean?

Look at Problem 11.10 on page 56 for an example of where this theorem is useful.

Definition A set like S (one which contains **all** of its limits) is called a **CLOSED** set.

lim inf and lim sup

Recall the definition of $\liminf s_n$ and $\limsup s_n$ of a sequence (s_n)
(visualize the picture!)

Theorem

12.2 Let (s_n) be any sequence of any nonzero reals. Then:

$$\liminf \left|\frac{s_{n+1}}{s_n}\right| \leq \liminf |s_n|^{1/n} \leq \limsup |s_n|^{1/n} \leq \limsup \left|\frac{s_{n+1}}{s_n}\right|$$

Theorem

12.3 If $\lim \left|\frac{s_{n+1}}{s_n}\right|$ exists and equals L, then $\lim |s_n|^{1/n}$ exists and equals L.

Next: Chapter 14 and Series!

Homework Assignment (Due Friday October 25)

§11, page 56: **11.4, 11.5** §12, page 60: **12.3, 12.14(a)**