Real Analysis

Math 310 MWF 10:30-11:25am
Fall 1996 127 Fowler

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#16 Friday (10/18/96)

SUMMARY Subsequences

READING pages 48-55

Warm-Up
Given the sequence $s_n := 3 + 2(-1)^n$ write down a definition for (t_k), a subsequence of (s_n) which takes on a **single** value (i.e. is constant).

Theorem
11.2 If A sequence (s_n) converges, then every subsequence (s_{n_k}) converges to the same limit
PROOF:

We are going to prove the famous Bolzano-Weierstrass Theorem, which is important later on when we are dealing with continuity of functions.
Bolzano-Weierstrass Theorem: Every bounded sequence has a convergent subsequence.

First we need to prove the following theorem.
Theorem
11.3 Every sequence (s_n) has a monotonic subsequence.
PROOF:

Bolzano-Weierstrass Theorem
11.5 Every bounded sequence has a convergent subsequence.
PROOF:

Corollary
11.4 Every sequence (s_n) has a monotonic subsequence which converges to $\limsup s_n$ and another one which converges to $\liminf s_n$

Definition
A *subsequential limit* of (s_n) is any real number, $+\infty$ or $-\infty$ which is the limit of some subsequence of (s_n)

Example
11.3 $a_n = \cos(n\pi/3)$

1. Give an example of a monotone subsequence

2. Write down the set of subsequential limits

3. Write down $\liminf a_n$ and $\limsup a_n$

4. Does a_n converge? diverge to $+\infty$? diverge to $-\infty$? N.O.T.A.?

5. Is a_n bounded?

Homework Assignment (Due Monday October 21)
§10, page 47-48: **10.1, 10.6 10.10**
§11, page 55: **11.2**