Real Analysis

Math 310 | MWF 10:30-11:25am
Fall 1996 | 127 Fowler

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#15 Wednesday (10/16/96)

SUMMARY Cauchy Sequences

READING pages 46-47

Warm-Up

(5-10 minutes) Recall the definition of a **Cauchy Sequence**

Example

Let's show that the sequence $s_n = 1/n$ is Cauchy.

Groupwork

Form three groups of two and in each group think of a convergent sequence.
Use the formal definition of what a Cauchy sequence is to show that your convergent sequence is Cauchy.

When we say that a sequence is Cauchy we have made quite a strong statement about the sequence. [Think about the **connections** between statements such as "This sequence is Cauchy." "This sequence is convergent." "This sequence is bounded."]

Theorem

10.9 All convergent sequences are Cauchy sequences.

Theorem

10.10 All Cauchy sequences are bounded.

Theorem

10.11 A sequence is convergent *if and only if* it is a Cauchy sequence.

Subsequences

Given a sequence (s_n) a *subsequence* of this sequence is a sequence (t_k) where for each k there is a positive integer n_k such that $t_k = s_{n_k}$. (Note that $n, k \in \mathbb{N}$ and and $n_1 < n_2 < \cdots < n_k < n_{k+1} < \cdots$)

Often the subset of $\mathbb{N}$ consisting of $\{n_1, n_2, n_3, \ldots\}$ is computed using a "selection function" denoted by $\sigma(k) = n_k$.

Example

Let (s_n) be the sequence defined by $s_n = n^2(-1)^n$

The positive elements of this sequence make up a subsequence.

- Write down the first 8 terms of (s_n)
- Write down the first 8 terms of (s_{n_k})
- Write down an expression for n_k in terms of k (i.e. $\sigma(k)$)
- Write down a definition of the k-th term of (s_{n_k})

How many subsequences does a sequence have? Think of one *other* subsequence of the above sequence.

Theorem

11.2 If A sequence (s_n) converges, then every subsequence (s_{n_k}) converges to the same limit

Proof:

Homework Assignment (Due Monday October 21)

§10, page 47-48: **10.1, 10.6 10.10**

§11, page 55: **11.2**