Real Analysis

Math 310 MWF 10:30-11:25am
Fall 1996 127 Fowler

Ron Buckmire ~ 402 Fowler ~ 259-2536 ~ `ron@oxy.edu` ~ Mon 1:30-3:30pm Wed & Thu 1:30-2:30pm

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Monday (10/07/96)

SUMMARY Introduction to $\lim sup$ and $\lim inf$

READING Pages 44-45

Warm-Up
(5-10 minutes) Review questions on the exam

Definitions
Let (s_n) be a sequence of real numbers.
We define

$$\limsup s_n = \lim_{N \to \infty} [\sup s_n : n > N]$$

and

$$\liminf s_n = \lim_{N \to \infty} [\inf s_n : n > N]$$

That's nice; but what do these definitions mean?

Maybe this discussion will help (also read page 44 of Ross):
Consider the expression in the square brackets. Really, $\sup s_n : n > N$ is an element of a new sequence. The book calls these terms v_N. As N is chosen to be larger and larger the terms v_N will get larger and larger and therefor one can find out what *their* limit is and call this number,$\lim_{N \to \infty} v_N = v$. This number v is known as the $\lim sup s_n$

A similar phenomena occurs if one thinks about the sequence consisting of $\inf s_n : n > N$ and the limit of this sequence is called $\liminf s_n$

We shall be using "the limit superior" and "the limit inferior" later on in the course as we discuss a major theorem about the Real Numbers.
(so this topic may seem weird but yes you will need to understand it later on in the course also)

Example
Consider the sequence $s_n = (-1)^n + \frac{1}{n}$

(a) What is $\lim s_n$?

(b) Is (s_n) bounded? monotone? convergent? divergent?

(c) What is $\liminf s_n$?

(d) What is $\limsup s_n$?

Homework Assignment (Due Monday October 14)
§10, page 55: 11.2