Real Analysis

Math 310 — MWF 10:30-11:25am
Fall 1996 — 127 Fowler

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SUMMARY Learning to Prove Convergence of Sequences

READING §7, §8

Warm-Up
(5-10 minutes) Let's review homework and answer any homework questions you may have.

Formal Proofs
We are going to learn how to write a formal proof of the convergence of a sequence to a particular limit by breaking down the process into steps.

Let's examine the sequence, (s_n), given by $s_n = \dfrac{1}{n^2}$

1. What is $\lim s_n$?

2. Write down the formal definition (see Definition 7.1, Ross) of this limit.

3. What must n be to satisfy the inequality in the formal definition if we let $\epsilon = 0.1$?

4. $\epsilon = 0.01$?

5. $\epsilon = 0.00001$?

6. What is the pattern you see here?

7. Suppose that I give you an arbitrary value for ϵ. Find what N must be in order for the inequality in the definition (2.) above to be satisfied.

8. From the above questions, we are now ready to present a **formal proof** of the convergence of this sequence.

FILL IN THE OUTLINE (The entire proof is in regular type; side notes are in *italics*):

Let $\epsilon > 0$. (*This arbitrary value of ϵ gives the $\forall \epsilon$ needed in the definition.*)

Let $N =$ _______________.
(*This is the N, which depends on ϵ, which must exist in the definition. Note that you found this above in 7. The whole idea is to find this first (show that it exists), and then present it in the formal proof.*)

Then we only need to show that $n > N \Rightarrow \left|\dfrac{1}{n^2}\right| < \epsilon$:

$$n > N$$

$$n >$$

(*Keep reversing what you did in 7 above and you'll end up at the right place!*)

Thus we've shown that

$$\forall\ \epsilon > 0\ \ \exists N = \qquad \ni n > N \Rightarrow \left|\frac{1}{n^2}\right| < \epsilon$$

and our proof is done. $\square$

Now try to prove the convergence of the following sequence on your own:

$$s_n = \frac{n+1}{2n-3}$$

1. What is $\lim s_n$?

2. Write down the formal definition (see Definition 7.1, Ross) of this limit. We did this on the previous worksheet (Friday).

7. Suppose that I give you an arbitrary value for ϵ. Find what N must be in order for the inequality in the definition above to be satisfied.

8. **FORMAL PROOF.** Use the outline of the proof on the previous page to guide you in presenting a formal proof here.

<u>**Homework**</u>
READ §8
DO : **8.1 (a),(c)**