Real Analysis

Math 310 — MWF 10:30-11:25am

Fall 1996 — 127 Fowler

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SUMMARY Exploring Sequences

READING §7, §8

Warm-Up
(5-10 minutes) Let's review homework and answer any homework questions you may have.

Sequence
A *sequence* is a function whose domain is of the form $\{n \in \mathbb{Z} : n \geq m\}$, where m is 0 or 1.

Today we will be exploring some sequences and limits of sequences.
On the computer, call up *Derive*. We will use various ways in *Derive* to look at sequences, (s_n), and think about convergence (limits) of these sequences.

Think of the sequence (s_n) as the function $s(n)$. To author a function, type/select **A** and then something like (the colon is important for DEFINING the function):

$$s(n) := \frac{1}{n^2}$$

To find the value of a particular element of this sequence, type/select **A** and then something like:

$$s(2)$$

To see a numerical value for this, you can either **S**implify it and then appro**X**imate the result, or you can just directly appro**X**imate it. Try to find a few more values for $s(n)$ for various values of $n \in \mathbb{N}$.

One way to find a limit is to just put in bigger and bigger values of n.
What's the limit of this example going to be?

$$\lim_{n\to\infty} s(n) = \lim_{n\to\infty} \frac{1}{n^2} =$$

Another way to find the limit in *Derive* is to use the built in **L**imit option under the **C**alculus menu. Type/select **C** and then **L**, enter the expression number (or you can retype the function again if you don't mind wasting time) of the function – you can use the up and down arrows to find the function again. It will ask you for the limiting variable (n in our case), and then the limit point (**inf** in our case). When you hit return it will write out the limit just like you're used to seeing. Notice when it asks for the limit point it also allows you to find **L**eft or **R**ight limits as well. Just use the **Tab** key to get over there if you wish to do a one-sided limit. Once you have the limit, you can once again find the numerical value by appro**X**imating it. Try the example we've been working on.

> A *Derive* note: Sometimes you just generate so many function definitions and other junk in the screen that you need to get rid of it all and start over again. To do this, type/select **T**ransfer, **C**lear, and **Y**es to get rid of all the expressions in there now. This will free up all the letters you previously used as functions.

Use *Derive*, or anything else, to figure out the limits of some of the sequences given in 7.1, 7.3 and all three of the sequences in 7.5. If they don't converge (there is no limit) state this fact. Make sure you play around with the sequences to get the feel for what they are doing — try $s(10)$, $s(100)$, $s(1000)$, etc. every once in a while before taking the limit.
Finish this exploration for the remaining problems on your own time.

Formal Definition of Convergence of a Sequence: A sequence (s_n) of real numbers is said to *converge* to the real number s provided that

$$\text{for each } \epsilon > 0 \text{ there exists a number } N \text{ such that } n > N \Rightarrow |s_n - s| < \epsilon.$$

(Note: We will have very different looking definitions for divergence since the symbols $\pm\infty \notin \mathbb{R}$. After we discuss this briefly, understand what this statement means and then rewrite the limit from the first page and a few other limits from 7.1,3,5 in this formal notation. Basically this means writing the formal definition with the sequence function filled in for s_n and the limiting value filled in for s.

The sequence $\left(\frac{1}{n^2}\right)$ converges to _____ provided that $\forall \epsilon > 0 \ \exists N \ni n > N \Rightarrow | \qquad\qquad | < \epsilon.$

Finish the exploration you began above for the sequences in 7.1, 7.3, and 7.5.
<u>**Homework**</u>
READ §7
DO : 7.4