Real Analysis

Math 310 MWF 10:30-11:25am
Fall 1996 127 Fowler

Ron Buckmire ~ 402 Fowler ~ 259-2536 ~ `ron@oxy.edu` ~ Mon 1:30-3:30pm Wed & Thu 1:30-2:30pm

---

Class #3 (09/11/96)

SUMMARY Really big numbers and really small numbers

READING §4, §5

Warm-Up
(5-10 minutes)
Show that $|a+b+c| \leq |a|+|b|+|c|$

Therefore, show that $\left|\sum_{i=1}^{N} x_i\right| \leq \sum_{i=1}^{N} |x_i|$

Definitions
Let S be a nonempty subset of $\mathbb{R}$.

Maximum If S contains an element s_0 such that $s_0 \geq s$ for all $s \in S$ then we call s_0 the *maximum of S* and write $s_0 =$`max S`

Minimum If S contains an element s_0 such that $s_0 \leq s$ for all $s \in S$ then we call s_0 the *maximum of S* and write $s_0 =$ `min S`

Upper Bound If there exists a number $M \in \mathbb{R}$ such that $s \leq M$ for all $s \in S$, then M is called the *upper bound of S* and the set S is said to be *bounded above*

Lower Bound If there exists a number $m \in \mathbb{R}$ such that $m \leq s$ for all $s \in S$, then m is called the *lower bound of S* and the set S is said to be *bounded below*

Bounded A set S is said to be *bounded* if it is bounded above and bounded below.

Supremum If S is bounded above and has a *least* upper bound, then we call this number the *supremum of S*, denoted by `sup S` or `lub S`

Infimum If S is bounded below and has a *greatest* lower bound, then we call this number the *infimum of S*, denoted by `inf S` or `glb S`

Examples
Determine whether the following sets are bounded above and/or bounded below.
Give the supremum/infimum if it exists.

- $[0, 1]$
- $2, 7$
- $1 - 1/3^n : n \in \mathbb{N}$
- $r \in \mathbb{Q} : r < 2$
- $r \in \mathbb{Q} : r^2 < 2$
- $\sin(n\pi/3) : n \in \mathbb{N}$

The Completeness Axiom
Every nonempty subset S of $\mathbb{R}$ that is *bounded above* has a *least upper bound*, called `sup S`.

The Archimedes Property
If $a > 0$ and $b > 0$, then there exists a positive integer n such that $na > b$

The Denseness of the Rationals
If $a, b \in \mathbb{R}$ and $a < b$ then there exists a rational number $r \in \mathbb{Q}$ such that $a < r < b$.

The Symbols $+\infty$ and $-\infty$

Homework
§4, page 19: Pick 10 sets from **4.1** different from ones done in class.