Real Analysis

Math 310 — MWF 10:30-11:25am
Fall 1996 — 127 Fowler

Ron Buckmire ~ 402 Fowler ~ 259-2536 ~ ron@oxy.edu ~ Mon 1:30-3:30pm Wed & Thu 1:30-2:30pm

Class #3

SUMMARY The Real Number System, $\mathbb{R}$

READING §3, §4

Warm-Up
(5-10 minutes)
Does there exist a rational number which is not algebraic? If so, give an example

Does there exist an algebraic number which is not rational? If so, give an example

Prove that your example is not a rational number (Use RZT)

The properties of $\mathbb{R}$

A1 $a+(b+c)=(a+b)+c$, for all $a,b,c \in \mathbb{R}$

A2 $a+b=b+a$, for all $a,b \in \mathbb{R}$

A3 $a+0=a$ for all $a \in \mathbb{R}$

A4 For each a, there is an element $-a$ such that $a+(-a)=0$

M1 $a(bc)=(ab)c$, for all $a,b,c \in \mathbb{R}$

M2 $ab=ba$ for all $a,b \in \mathbb{R}$

M3 $a \cdot 1 = a$ for all $a \in \mathbb{R}$

M4 For each $a \neq 0$, ther is an element a^{-1} such that $a \cdot a^{-1} = 1$

DL $a \cdot (b+c) = a \cdot b + a \cdot c$

Definition: A set which has at least one element and obeys the above properties (known as the *field axioms*) is said to be a **field**.

Which of the following are fields?
(Determine which of the field axioms are violated by which sets of numbers)

- Natural numbers, $\mathbb{N}$
- Integers, $\mathbb{Z}$
- Rational numbers, $\mathbb{Q}$
- Real numbers, $\mathbb{R}$

$\mathbb{Q}$ and $\mathbb{R}$ are known as **ordered fields** because they obey the following *ordering rules* in addition to the field axioms:

O1 Given a and b, either $a \leq b$ or $b \leq a$

O2 If $a \leq b$ and $b \leq a$, then $a = b$

O3 If $a \leq b$ and $b \leq c$ then $a \leq c$

O4 If $a \leq b$ then $a + c \leq b + c$

O5 If $a \leq b$ and $0 \leq c$ then $ac \leq bc$

There are a number of consequences to satisfying the **field axioms** and the **ordering rules**. These are compiled and listed as Theorems 3.1 and 3.2 on pages 11 and 12 of the text. Review these pages carefully.

The real numbers are being developed from the rational numbers. In fact one of the big questions that you should be facing right now is: How do we know $\mathbb{Q}$ and $\mathbb{R}$ are different sets?

<u>Definitions</u>
We define the *absolute value* of a to be $|a|$ and $|a| = a$ if $a \geq 0$ and $|a| = -a$, if $a < 0$

We define *the distance between two numbers* a and b to be $\texttt{dist}(a, b)$ and $\texttt{dist}(a, b) = |a - b|$
The Triangle Inequality
A rather famous result is $|a + b| \leq |a| + |b|$ (for any $a, b \in \mathbb{R}$)

<u>Homework</u>
§3, page 14: **3.3, 3.4** and **3.5**