Real Analysis

Math 310 — MWF 10:30-11:25am

Fall 1996 — 127 Fowler

Ron Buckmire ~ 402 Fowler ~ 259-2536 ~ `ron@oxy.edu` ~ Mon 1:30-3:30pm Wed & Thu 1:30-2:30pm

Class #2

SUMMARY Natural, Rational and Algebraic Numbers

READING §1, §2 (pp 1-9)

Warm-Up
What are the *Peano Postulates* (a.k.a. Peano Axioms)?
Write down as many of them as you can remember. (How many of them are there?)

$\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$

One of the main goals of this course is to understand the construction of the real numbers. But before we get there we will be dealing with other types of numbers first.

Give three examples of each type of number below:

- Natural
- Integer
- Rational
- Algebraic
- Real

What's an **algebraic number**?

Formal Definition: A number is called an *algebraic number* if it satisfies a polynomial equation

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0 = 0$$

where the coefficients $a_0, a_1, a_2, \ldots, a_n$ are integers, $a_n \neq 0$ and $n \geq 1$.
What polynomial does the number $b = \sqrt{4 - \sqrt[3]{3}}$ satisfy? Is b an algebraic number?

Suppose we want to check if a number is **rational** or not. What can we do?

Rational Zeros Theorem: Suppose that $a_0, a_1, \ldots, a_n$ are integers and that r is a rational number satisfying the polynomial equation

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0 = 0$$

where $a_0 \neq 0$, $a_n \neq 0$ and $n \geq 1$. Write $r = p/q$ where p, q are integers having no common factors and $q \neq 0$. Then q divides a_n and p divides a_0.

What does this mean?

Example

1. Using RZT, show that $\sqrt{2}$ cannot represent a rational number.

2. Using RZT, show that $\sqrt[3]{6}$ cannot represent a rational number.

At Home

DO: §2, page 9: **2.3, 2.6**

READ: §3, pages 10-14