(Mock) Test: Real Analysis

Assigned: Wed Sep 25 **Due: Fri Sep 27**

Name: **1 hour**

Directions: This is a closed-book mock exam. The idea is to give you some sense of the type of question which will appear on Exam 1 of this course. You should be able to complete this test in **1 hour**. You may not use any books, notes, worksheets, graded homework sets to assist you. You may not discuss any aspect of this exam with any other person except **me**, the instructor. This exam must be handed in by noon Friday September 27, to the math department secretary Caroline in Fowler 221.

Show all your work.

No.	Score	Maximum
1		30
2		30
3		15
4		15
5		10
Total		100

Show complete work in the space provided on the following pages. Although the proofs are only worth 40% of the total grade, you will probably want to spend a majority of your time on these. I've given you plenty of room to do some preliminary work on the proofs and to present your formal proof. Make sure your formal proof is clearly indicated!

1. *[30 pts. total]* Complete the following ((a)-(c) require FORMAL definitions):

(a) A sequence (s_n) is bounded above if ...

(b) A sequence (s_n) of real numbers is said to converge to s (i.e. $\lim s_n = s$) provided that ...

(c) A sequence (s_n) diverges to $-\infty$ (i.e. $\lim s_n = -\infty$) provided that ...

(d) Let (s_n) be a sequence in $\mathbb{R}$. We say that a limit of the sequence *exists* if ...

(e) Give the following limits, or write DNE if the limit does not exist. No justification required:

(i) $\lim(-1)^n =$ (ii) $\lim \dfrac{(-1)^n}{n} =$ (iii) $\lim \dfrac{n^2}{1-n} =$

2 (30 pts. / 6 pts. ea.) TRUE/FALSE. Circle one for each statement below. To be true, the statement must ALWAYS be true.

(a) If (s_n) converges, then (s_n) is bounded.

TRUE FALSE

(b) $|-x| = x$.

TRUE FALSE

(c) If $A = \{r \in \mathbb{Q} : 0 < r < \sqrt{2}\}$, then $\sup A$ does not exist because $\sqrt{2} \notin A$.

TRUE FALSE

(d) $\lim s_n = 0$ if and only if $\lim |s_n| = 0$.

TRUE FALSE

(e) If S is a set of real numbers, then $\inf S \leq \sup S$.

TRUE FALSE

3 (15 pts.) Prove directly from the definition of convergence:

$$\lim \frac{n+1}{n^3+14} = 0.$$

4 (15 pts.) Prove that

$$2^n > n^2, \qquad \forall n \in \mathbb{N}.$$

5 (10 pts.) Prove directly from the definition of divergence:

$$\textit{If } \lim s_n = +\infty \textit{ and } k > 0, \textit{ then } \lim(ks_n) = +\infty.$$