Report on Test 1

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Point Distribution (N=17)

Range	90+	86+	82+	73+	70+	67+	63+	70+	68+	63+	50+	50-
Grade	A	A-	B+	B	B-	C+	C	C-	D+	D+	D-	F
Frequency	4	3	0	8	0	1	0	0	0	0	0	1

Comments

Overall Overall performance was satisfactory. I was quite surprised (but pleased) that almost half of the class (7/17) earned an A on the exam. The mean score was 80.0. The median score was 78.

#1 Equations of Lines and Planes. Just use the formulas for parametric equations. An interesting thing to recall is the Cartesian form of those equations. In $\mathcal{R}^n$ you'll need $n-1$ Cartesian equations to describe a line, and $n-k$ equations to describe a k-plane. The cartesian equation(s) of the line is $y = -1$ and $z = 3x/2 + 1/2$. It turns out the equation of the plane is $-3x + y + z = 0$ since $(-3, 1, 1)$ is orthogonal (take the cross-product) to $(-1, -1, -1)$ and $(1, -1, 2)$.

#2 Point Sets in the Plane. 0^0 is not well-defined. Thus the point (0,0) can not be in the domain of the function x^y. The complement of an open set is a closed set. If a set is open, it is not closed and if it is closed it is not open! Thus, A was CLOSED (NOT OPEN) and NOT a DOMAIN. B was CLOSED (NOT OPEN) and NOT a DOMAIN. C and D were both NOT CLOSED and NOT OPEN since it only contained part of their boundaries. E was the only OPEN (NOT CLOSED) set. C, D and E were all domains since they do not include the origin $(0, 0)$ where the function is undefined. The domain of x^y must be the set of all points where x is greater than or equal to 0, as long as you don't raise 0 to a negative power.

#3 Limits. This question was to check whether you really understood the nature of limits as sequences of numbers. **(a)** $\lim\limits_{x\to 0^+} x^{\alpha x} = \lim\limits_{x\to 0^+} (x^x)^\alpha = 1^\alpha = 1$. **(b)** When you take the limit along the x-axis, $y = 0$ so you are looking at a sequence of numbers x^0 for increasingly smaller positive values of x: $.1^0, .01^0, .001^0, \cdots = 1, 1, 1, 1, 1, \ldots, = 1$. **(c)** When you take the limit along the y-axis, $x = 0$ so you are now looking at a sequence of numbers 0^y for increasinginly smaller positive values of y: $0^{0.1}, 0^{0.01}, 0^{0.000001}, \cdots = 0, 0, 0, 0, 0, 0, \ldots = 0$. **(d)** Since the limit along different paths produces different results, the multivariable limit to the origin does not exist.

#4 Gradient Operator (a) Remember the gradient operator is a vector, and that $\ln(1) = 0$. When $f(x, y) = x^y$, $f_x = yx^{y-1}$ using the "derivative power rule" and $f_y = x^y \ln(x)$ using the "derivative exponent rule." **(b)** The directional derivative must always involve a unit vector, so you have to normalize (1,1) to be $(1/\sqrt{2}, 1/\sqrt{2})$ BEFORE taking the dot product with $\vec{\nabla} f(x, y)$ **(c)** The function x^y is a scalar which you know at $f(1, 1) = 1^1 = 1$, so $1.1^1.2 \approx f(1, 1) + f_x(1, 1)(x - 1) + f_y(1, 1)(y - 1)$. Thus the Tangent Approximation is $T(x, y) = x$ at (1,1).

#5 Partial Differential Equations Just take two partial derivatives and show that $f_{xx} + f_{yy} \neq 0$ for all (x,y) so the function is not harmonic. $f_{xx} = y(y-1)x^{y-1}$ and $f_{yy} = x^y \ln(x) \ln(x) = x^y (\ln(x))^2$. Clearly the function x^y is not harmonic at all points in its domain, though it does satisfy Laplace's Eqaution at the one point $(1, 1)$.