Test 1: Multivariable Calculus

Math 224 — Friday February 27 2004
Ron Buckmire — 2:30pm-3:30pm

Name: _____

Directions: Read *all* problems first before answering any of them. Questions 2-5 are all related, but different. This is a one hour, open-notes, open book, test. **No calculators.** You must show all relevant work to support your answers. Use complete English sentences and CLEARLY indicate your final answers to be graded from your "scratch work."

No.	Score	Maximum
1		20
2		20
3		20
4		30
5		10
Extra Credit		5
Total		**100**

1. (*20 points.*) **Equations of Planes and Lines.**
a. (*10 points*) Find the equation of the line that goes through the points **A** $(-1, -1, -1)$ and **B** $(1, -1, 2)$.

b. (*10 points*) Find the equation of the plane that goes through the points **A** and **B** and a third point **C** $(0, 0, 0)$

2. (*20 points.*) **Point Sets in the Plane.**
Consider the following sets in the $\mathbb{R}^2$. Characterize the sets (by CIRCLING THE WORD) as either OPEN, CLOSED, NOT OPEN or NOT CLOSED. In addition, indicate which sets *could* be a domain for the function $f(x, y) = x^y$. Some sets may have one more than one chracterization. In other words, **circle as many words for each set as you think apply**.

NOTE: There is also space to explain your answers.

a. (*4 points*) $A = \{(x, y) : x \geq 0\}$
OPEN NOT OPEN CLOSED NOT CLOSED DOMAIN

b. (*4 points*) $B = \{(x, y) : x \geq 0 \cap y \geq 0\}$
OPEN NOT OPEN CLOSED NOT CLOSED DOMAIN

c. (*4 points*) $C = \{(x, y) : x \geq 0 \cap y > 0\}$
OPEN NOT OPEN CLOSED NOT CLOSED DOMAIN

d. (*4 points*) $D = \{(x, y) : x > 0 \cap y \geq 0\}$
OPEN NOT OPEN CLOSED NOT CLOSED DOMAIN

e. (*4 points*) $E = \{(x, y) : x > 0 \cap y > 0\}$
OPEN NOT OPEN CLOSED NOT CLOSED DOMAIN

EXTRA CREDIT
(*5 points*) Write down a definition of the set of points in $\mathbb{R}^2$ which make up the largest possible domain of $f(x, y) = x^y$. Give a small sketch of your set.

3. (*20 points.*) **Limits of Multivariable Functions.**
Let's consider the function $f(x,y) = x^y$. (Note: $\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{x\ln(x)} = 1$.)

a. (*6 points*) Show that along the path $y = \alpha x$, $\lim_{(x,y)\to(0,0)} x^y = 1$ for $\alpha \in \mathbb{R}$

b.(*5 points*) Using a (one-sided) path along the x-axis, evaluate $\lim_{(x,y)\to(0,0)} x^y$

c. (*5 points*) Using a (one-sided) path along the y-axis, evaluate $\lim_{(x,y)\to(0,0)} x^y$

d. (*4 points*) What is the value of $\lim_{(x,y)\to(0,0)} x^y$?

4. (*30 points.*) **Gradient Operator, Directional Derivative, Tangent Approximation.**
Consider $f(x, y) = x^y$.

a. (*10 points*) Evaluate $\vec{\nabla} f(1, 1)$.

b. (*10 points*) Compute the general directional derivative of f in the **direction** $(1, 1)$.

c. (*10 points*) Obtain an estimate of $1.1^{1.2}$ using an appropriate tangent approximation to $f(x, y) = x^y$.

5. (*10 points.*) **Partial Derivatives.**
Recall that Laplace's Equation is $\phi_{xx} + \phi_{yy} = 0$. A function $f(x, y)$ which satisfies Laplace's Equation is called **harmonic.** Is $f(x, y) = x^y$ a harmonic function? **Prove your answer.**