

DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\sin u] = (\cos u)u'$
12. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
13. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
14. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
15. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
16. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
17. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$
18. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$
19. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
20. $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1+u^2}$
21. $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$
22. $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$

Basic Integration Formulas

1. $\int kf(u) du = k \int f(u) du$
2. $\int [f(u) \pm g(u)] du = \int f(u) du \pm \int g(u) du$
3. $\int du = u + C$
4. $\int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
5. $\int \frac{du}{u} = \ln|u| + C$
6. $\int e^u du = e^u + C$
7. $\int \sin u du = -\cos u + C$
8. $\int \cos u du = \sin u + C$
9. $\int \tan u du = -\ln|\cos u| + C$
10. $\int \cot u du = \ln|\sin u| + C$
11. $\int \sec u du = \ln|\sec u + \tan u| + C$
12. $\int \csc u du = -\ln|\csc u + \cot u| + C$
13. $\int \sec^2 u du = \tan u + C$
14. $\int \csc^2 u du = -\cot u + C$
15. $\int \sec u \tan u du = \sec u + C$
16. $\int \csc u \cot u du = -\csc u + C$
17. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$
18. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$
19. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

Integration Tables

C

Forms Involving u^n

1. $\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$

2. $\int \frac{1}{u} du = \ln|u| + C$

Forms Involving $a + bu$

3. $\int \frac{u}{a + bu} du = \frac{1}{b^2}(bu - a \ln|a + bu|) + C$

4. $\int \frac{u}{(a + bu)^2} du = \frac{1}{b^2}\left(\frac{a}{a + bu} + \ln|a + bu|\right) + C$

5. $\int \frac{u}{(a + bu)^n} du = \frac{1}{b^2}\left[\frac{-1}{(n-2)(a + bu)^{n-2}} + \frac{a}{(n-1)(a + bu)^{n-1}}\right] + C, n \neq 1, 2$

6. $\int \frac{u^2}{a + bu} du = \frac{1}{b^3}\left[-\frac{bu}{2}(2a - bu) + a^2 \ln|a + bu|\right] + C$

7. $\int \frac{u^2}{(a + bu)^2} du = \frac{1}{b^3}\left(bu - \frac{a^2}{a + bu} - 2a \ln|a + bu|\right) + C$

8. $\int \frac{u^2}{(a + bu)^3} du = \frac{1}{b^3}\left[\frac{2a}{a + bu} - \frac{a^2}{2(a + bu)^2} + \ln|a + bu|\right] + C$

9. $\int \frac{u^2}{(a + bu)^n} du = \frac{1}{b^3}\left[\frac{-1}{(n-3)(a + bu)^{n-3}} + \frac{2a}{(n-2)(a + bu)^{n-2}} - \frac{a^2}{(n-1)(a + bu)^{n-1}}\right] + C, n \neq 1, 2, 3$

10. $\int \frac{1}{u(a + bu)} du = \frac{1}{a} \ln\left|\frac{u}{a + bu}\right| + C$

11. $\int \frac{1}{u(a + bu)^2} du = \frac{1}{a}\left(\frac{1}{a + bu} + \frac{1}{a} \ln\left|\frac{u}{a + bu}\right|\right) + C$

12. $\int \frac{1}{u^2(a + bu)} du = -\frac{1}{a}\left(\frac{1}{u} + \frac{b}{a} \ln\left|\frac{u}{a + bu}\right|\right) + C$

13. $\int \frac{1}{u^2(a + bu)^2} du = -\frac{1}{a^2}\left[\frac{a + 2bu}{u(a + bu)} + \frac{2b}{a} \ln\left|\frac{u}{a + bu}\right|\right] + C$

Forms Involving $a + bu + cu^2$, $b^2 \neq 4ac$

$$14. \int \frac{1}{a + bu + cu^2} du = \begin{cases} \frac{2}{\sqrt{4ac - b^2}} \arctan \frac{2cu + b}{\sqrt{4ac - b^2}} + C, & b^2 < 4ac \\ \frac{1}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2cu + b - \sqrt{b^2 - 4ac}}{2cu + b + \sqrt{b^2 - 4ac}} \right| + C, & b^2 > 4ac \end{cases}$$

$$15. \int \frac{u}{a + bu + cu^2} du = \frac{1}{2c} \left(\ln|a + bu + cu^2| - b \int \frac{1}{a + bu + cu^2} du \right)$$

Forms Involving $\sqrt{a + bu}$

$$16. \int u^n \sqrt{a + bu} du = \frac{2}{b(2n + 3)} \left[u^n(a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} du \right]$$

$$17. \int \frac{1}{u \sqrt{a + bu}} du = \begin{cases} \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, & a > 0 \\ \frac{2}{\sqrt{-a}} \arctan \sqrt{\frac{a + bu}{-a}} + C, & a < 0 \end{cases}$$

$$18. \int \frac{1}{u^n \sqrt{a + bu}} du = \frac{-1}{a(n-1)} \left[\frac{\sqrt{a + bu}}{u^{n-1}} + \frac{(2n-3)b}{2} \int \frac{1}{u^{n-1} \sqrt{a + bu}} du \right], n \neq 1$$

$$19. \int \frac{\sqrt{a + bu}}{u} du = 2\sqrt{a + bu} + a \int \frac{1}{u \sqrt{a + bu}} du$$

$$20. \int \frac{\sqrt{a + bu}}{u^n} du = \frac{-1}{a(n-1)} \left[\frac{(a + bu)^{3/2}}{u^{n-1}} + \frac{(2n-5)b}{2} \int \frac{\sqrt{a + bu}}{u^{n-1}} du \right], n \neq 1$$

$$21. \int \frac{u}{\sqrt{a + bu}} du = \frac{-2(2a - bu)}{3b^2} \sqrt{a + bu} + C$$

$$22. \int \frac{u^n}{\sqrt{a + bu}} du = \frac{2}{(2n+1)b} \left(u^n \sqrt{a + bu} - na \int \frac{u^{n-1}}{\sqrt{a + bu}} du \right)$$

Forms Involving $a^2 \pm u^2$, $a > 0$

$$23. \int \frac{1}{a^2 + u^2} du = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$24. \int \frac{1}{u^2 - a^2} du = - \int \frac{1}{a^2 - u^2} du = \frac{1}{2a} \ln \left| \frac{u - a}{u + a} \right| + C$$

$$25. \int \frac{1}{(a^2 \pm u^2)^n} du = \frac{1}{2a^2(n-1)} \left[\frac{u}{(a^2 \pm u^2)^{n-1}} + (2n-3) \int \frac{1}{(a^2 \pm u^2)^{n-1}} du \right], n \neq 1$$

Forms Involving $\sqrt{u^2 \pm a^2}$, $a > 0$

$$26. \int \sqrt{u^2 \pm a^2} du = \frac{1}{2} (u \sqrt{u^2 \pm a^2} \pm a^2 \ln|u + \sqrt{u^2 \pm a^2}|) + C$$

$$27. \int u^2 \sqrt{u^2 \pm a^2} du = \frac{1}{8} [u(2u^2 \pm a^2) \sqrt{u^2 \pm a^2} - a^4 \ln|u + \sqrt{u^2 \pm a^2}|] + C$$

$$28. \int \frac{\sqrt{u^2 + a^2}}{u} du = \sqrt{u^2 + a^2} - a \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

$$29. \int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \operatorname{arcsec} \frac{|u|}{a} + C$$

$$30. \int \frac{\sqrt{u^2 \pm a^2}}{u^2} du = \frac{-\sqrt{u^2 \pm a^2}}{u} + \ln|u + \sqrt{u^2 \pm a^2}| + C$$

$$31. \int \frac{1}{\sqrt{u^2 \pm a^2}} du = \ln|u + \sqrt{u^2 \pm a^2}| + C$$

$$32. \int \frac{1}{u\sqrt{u^2 + a^2}} du = \frac{-1}{a} \ln \left| \frac{a + \sqrt{u^2 + a^2}}{u} \right| + C$$

$$33. \int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$$

$$34. \int \frac{u^2}{\sqrt{u^2 \pm a^2}} du = \frac{1}{2} (u\sqrt{u^2 \pm a^2} \mp a^2 \ln|u + \sqrt{u^2 \pm a^2}|) + C$$

$$35. \int \frac{1}{u^2 \sqrt{u^2 \pm a^2}} du = \mp \frac{\sqrt{u^2 \pm a^2}}{a^2 u} + C$$

$$36. \int \frac{1}{(u^2 \pm a^2)^{3/2}} du = \frac{\pm u}{a^2 \sqrt{u^2 \pm a^2}} + C$$

Forms Involving $\sqrt{a^2 - u^2}$, $a > 0$

$$37. \int \sqrt{a^2 - u^2} du = \frac{1}{2} \left(u\sqrt{a^2 - u^2} + a^2 \arcsin \frac{u}{a} \right) + C$$

$$38. \int u^2 \sqrt{a^2 - u^2} du = \frac{1}{8} \left[u(2u^2 - a^2) \sqrt{a^2 - u^2} + a^4 \arcsin \frac{u}{a} \right] + C$$

$$39. \int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$40. \int \frac{\sqrt{a^2 - u^2}}{u^2} du = \frac{-\sqrt{a^2 - u^2}}{u} - \arcsin \frac{u}{a} + C$$

$$41. \int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin \frac{u}{a} + C$$

$$42. \int \frac{1}{u\sqrt{a^2 - u^2}} du = \frac{-1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$43. \int \frac{u^2}{\sqrt{a^2 - u^2}} du = \frac{1}{2} \left(-u\sqrt{a^2 - u^2} + a^2 \arcsin \frac{u}{a} \right) + C$$

$$44. \int \frac{1}{u^2 \sqrt{a^2 - u^2}} du = \frac{-\sqrt{a^2 - u^2}}{a^2 u} + C$$

$$45. \int \frac{1}{(a^2 - u^2)^{3/2}} du = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$$

Forms Involving $\sin u$ or $\cos u$

$$46. \int \sin u \, du = -\cos u + C$$

$$47. \int \cos u \, du = \sin u + C$$

$$48. \int \sin^2 u \, du = \frac{1}{2}(u - \sin u \cos u) + C$$

$$49. \int \cos^2 u \, du = \frac{1}{2}(u + \sin u \cos u) + C$$

$$50. \int \sin^n u \, du = -\frac{\sin^{n-1} u \cos u}{n} + \frac{n-1}{n} \int \sin^{n-2} u \, du$$

$$51. \int \cos^n u \, du = \frac{\cos^{n-1} u \sin u}{n} + \frac{n-1}{n} \int \cos^{n-2} u \, du$$

$$52. \int u \sin u \, du = \sin u - u \cos u + C$$

$$53. \int u \cos u \, du = \cos u + u \sin u + C$$

$$54. \int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$$

$$55. \int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$$

$$56. \int \frac{1}{1 \pm \sin u} \, du = \tan u \mp \sec u + C$$

$$57. \int \frac{1}{1 \pm \cos u} \, du = -\cot u \pm \csc u + C$$

$$58. \int \frac{1}{\sin u \cos u} \, du = \ln|\tan u| + C$$

Forms Involving $\tan u$, $\cot u$, $\sec u$, $\csc u$

$$59. \int \tan u \, du = -\ln|\cos u| + C$$

$$60. \int \cot u \, du = \ln|\sin u| + C$$

$$61. \int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$62. \int \csc u \, du = \ln|\csc u - \cot u| + C$$

63. $\int \tan^2 u \, du = -u + \tan u + C$
64. $\int \cot^2 u \, du = -u - \cot u + C$
65. $\int \sec^2 u \, du = \tan u + C$
66. $\int \csc^2 u \, du = -\cot u + C$
67. $\int \tan^n u \, du = \frac{\tan^{n-1} u}{n-1} - \int \tan^{n-2} u \, du, \, n \neq 1$
68. $\int \cot^n u \, du = -\frac{\cot^{n-1} u}{n-1} - \int (\cot^{n-2} u) \, du, \, n \neq 1$
69. $\int \sec^n u \, du = \frac{\sec^{n-2} u \tan u}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} u \, du, \, n \neq 1$
70. $\int \csc^n u \, du = -\frac{\csc^{n-2} u \cot u}{n-1} + \frac{n-2}{n-1} \int \csc^{n-2} u \, du, \, n \neq 1$
71. $\int \frac{1}{1 \pm \tan u} \, du = \frac{1}{2}(u \pm \ln|\cos u \pm \sin u|) + C$
72. $\int \frac{1}{1 \pm \cot u} \, du = \frac{1}{2}(u \mp \ln|\sin u \pm \cos u|) + C$
73. $\int \frac{1}{1 \pm \sec u} \, du = u + \cot u \mp \csc u + C$
74. $\int \frac{1}{1 \pm \csc u} \, du = u - \tan u \pm \sec u + C$

Forms Involving Inverse Trigonometric Functions

75. $\int \arcsin u \, du = u \arcsin u + \sqrt{1-u^2} + C$
76. $\int \arccos u \, du = u \arccos u - \sqrt{1-u^2} + C$
77. $\int \arctan u \, du = u \arctan u - \ln \sqrt{1+u^2} + C$
78. $\int \operatorname{arccot} u \, du = u \operatorname{arccot} u + \ln \sqrt{1+u^2} + C$
79. $\int \operatorname{arcsec} u \, du = u \operatorname{arcsec} u - \ln|u + \sqrt{u^2-1}| + C$
80. $\int \operatorname{arccsc} u \, du = u \operatorname{arccsc} u + \ln|u + \sqrt{u^2-1}| + C$

Forms Involving e^u

$$81. \int e^u du = e^u + C$$

$$82. \int ue^u du = (u - 1)e^u + C$$

$$83. \int u^n e^u du = u^n e^u - n \int u^{n-1} e^u du$$

$$84. \int \frac{1}{1 + e^u} du = u - \ln(1 + e^u) + C$$

$$85. \int e^{au} \sin bu du = \frac{e^{au}}{a^2 + b^2} (a \sin bu - b \cos bu) + C$$

$$86. \int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \sin bu) + C$$

Forms Involving $\ln u$

$$87. \int \ln u du = u(-1 + \ln u) + C$$

$$88. \int u \ln u du = \frac{u^2}{4} (-1 + 2 \ln u) + C$$

$$89. \int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2} [-1 + (n+1) \ln u] + C, \quad n \neq -1$$

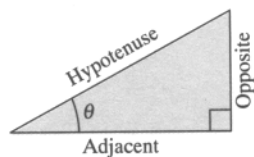
$$90. \int (\ln u)^2 du = u [2 - 2 \ln u + (\ln u)^2] + C$$

$$91. \int (\ln u)^n du = u(\ln u)^n - n \int (\ln u)^{n-1} du$$

TRIGONOMETRY

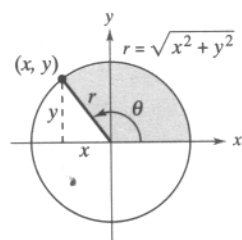
Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

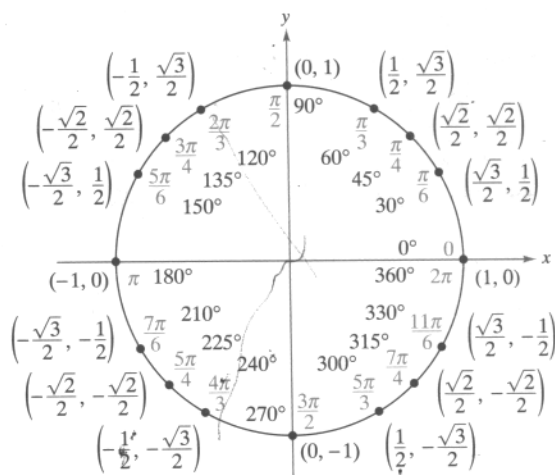


$$\begin{aligned}\sin \theta &= \frac{\text{opp}}{\text{hyp}} & \csc \theta &= \frac{\text{hyp}}{\text{opp}} \\ \cos \theta &= \frac{\text{adj}}{\text{hyp}} & \sec \theta &= \frac{\text{hyp}}{\text{adj}} \\ \tan \theta &= \frac{\text{opp}}{\text{adj}} & \cot \theta &= \frac{\text{adj}}{\text{opp}}\end{aligned}$$

Circular function definitions, where θ is any angle.



$$\begin{aligned}\sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$



Reciprocal Identities

$$\begin{aligned}\sin x &= \frac{1}{\csc x} & \sec x &= \frac{1}{\cos x} & \tan x &= \frac{1}{\cot x} \\ \csc x &= \frac{1}{\sin x} & \cos x &= \frac{1}{\sec x} & \cot x &= \frac{1}{\tan x}\end{aligned}$$

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1 \\ 1 + \tan^2 x &= \sec^2 x & 1 + \cot^2 x &= \csc^2 x\end{aligned}$$

Cofunction Identities

$$\begin{aligned}\sin\left(\frac{\pi}{2} - x\right) &= \cos x & \cos\left(\frac{\pi}{2} - x\right) &= \sin x \\ \csc\left(\frac{\pi}{2} - x\right) &= \sec x & \tan\left(\frac{\pi}{2} - x\right) &= \cot x \\ \sec\left(\frac{\pi}{2} - x\right) &= \csc x & \cot\left(\frac{\pi}{2} - x\right) &= \tan x\end{aligned}$$

Reduction Formulas

$$\begin{aligned}\sin(-x) &= -\sin x & \cos(-x) &= \cos x \\ \csc(-x) &= -\csc x & \tan(-x) &= -\tan x \\ \sec(-x) &= \sec x & \cot(-x) &= -\cot x\end{aligned}$$

Sum and Difference Formulas

$$\begin{aligned}\sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}\end{aligned}$$

Double-Angle Formulas

$$\begin{aligned}\sin 2u &= 2 \sin u \cos u \\ \cos 2u &= \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \\ \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u}\end{aligned}$$

Power-Reducing Formulas

$$\begin{aligned}\sin^2 u &= \frac{1 - \cos 2u}{2} \\ \cos^2 u &= \frac{1 + \cos 2u}{2} \\ \tan^2 u &= \frac{1 - \cos 2u}{1 + \cos 2u}\end{aligned}$$

Sum-to-Product Formulas

$$\begin{aligned}\sin u + \sin v &= 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \sin u - \sin v &= 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \\ \cos u + \cos v &= 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \cos u - \cos v &= -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)\end{aligned}$$

Product-to-Sum Formulas

$$\begin{aligned}\sin u \sin v &= \frac{1}{2} [\cos(u-v) - \cos(u+v)] \\ \cos u \cos v &= \frac{1}{2} [\cos(u-v) + \cos(u+v)] \\ \sin u \cos v &= \frac{1}{2} [\sin(u+v) + \sin(u-v)] \\ \cos u \sin v &= \frac{1}{2} [\sin(u+v) - \sin(u-v)]\end{aligned}$$