Linear Systems

Math 214 Spring 2008 — Fowler 309 MWF 9:30 am - 10:25 am

 — http://faculty.oxy.edu/ron/math/214/08/

Class 29

TITLE Wrapping it all together!
CURRENT READING Poole

Summary
The text uses the repetitive theme of adding statements to the Fundamental Theorem of Invertible Matrices as a theme. We'll look at the **final** version.

Theorem 7.19

The Fundamental Theorem of Invertible Matrices (Final Version). Let A be a $n \times n$ matrix and let $T : \mathcal{V} \to \mathcal{W}$ be a linear transformation whose standard matrix is A. Each of the following statements is equivalent:

(a) A is invertible.
(b) $A\vec{x} = \vec{b}$ has a unique solution for every $\vec{b}$ in $\mathbb{R}^n$.
(c) $A\vec{x} = \vec{0}$ has only the trivial solution.
(d) The reduced row echelon form of A, rref(A), is I_n.
(e) A is a product of elementary matrices.
(f) rank$(A) = n$.
(g) nullity$(A) = 0$.
(h) The column vectors of A are linearly independent.
(i) The column vectors of A span $\mathbb{R}^n$.
(j) The column vectors of A form a basis for $\mathbb{R}^n$.
(k) The row vectors of A are linearly independent.
(l) The row vectors of A span $\mathbb{R}^n$.
(m) The row vectors of A form a basis for $\mathbb{R}^n$.
(n) The determinant of A is not equal to zero.
(o) 0 is not an eigenvalue of A.
(p) T is invertible.
$\vdots \qquad \vdots$
(s) $\ker(T) = \left\{\vec{0}\right\}$.
(t) range$(T) = \mathcal{W}$.

Note that the book's version has some more concepts dealing with Linear Transformations and Changes of Basis that we did not discuss in this version of the course. The range of a linear transformation is the set of all possible outputs given its domain and the kernel (denoted `ker()`) is the set of points which the given transformation maps to the zero vector. You cna think of it as the equivalent of "the nullspace of a linear tranformation."