Linear Systems

Math 214 Spring 2008 Fowler 309 MWF 9:30 am - 10:25 am

 http://faculty.oxy.edu/ron/math/214/08/

Class 14: **Monday February 25**

SUMMARY LU Decomposition and Permutation Matrices
CURRENT READING Poole 3.4

Summary
We have found that we could (sometimes) find a matrix A^{-1} which converted A into the identity matrix I, on multiplication. We had also previously shown that we could find a series of E_{ij} matrices which when multiplied in sequence would convert A into an upper triangular matrix. Today we will attempt to regularize this process and show we can use these ideas to factor a matrix A into the product of a lower triangular matrix L and upper triangular matrix U.

Homework Assignment
*HW # 14: Section 3.4: 1,***2,3***,7,***8,9***,10,13,19,20. EXTRA CREDIT 26.*

1. LU Factorization

Consider the matrix $A = \begin{bmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{bmatrix}$

Can you show that this can be converted into upper triangular form by multiplying by a series of matrices E_{21}, E_{31} and E_{32}?

$U = \begin{bmatrix} 2 & 4 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{bmatrix}$

We have that $E_{32}E_{31}E_{21}A = U$

This means that

$A = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}U = L \cdot U$

Write down the elimination matrices you used to convert A into U

Write down the INVERSE of each of these three matrices.

Note that all of these matrices E_{21}, E_{31}, E_{32}, E_{21}^{-1}, E_{31}^{-1} and E_{32}^{-1} are **all** LOWER TRIANGULAR. Compute the product $E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}$. It is ALSO lower triangular. We call it L.

Now check that the product of L and U is, in fact, A.

The Point
We can use LU factorization to assist us in solving $A\vec{x} = \vec{b}$
$LU\vec{x} = b$ becomes the two systems of $L\vec{y} = \vec{b}$ and $U\vec{x} = \vec{y}$

Solving a lower triangular system and then an upper triangular system is much more computationally efficient than finding A^{-1}.

Let's do an example with $\vec{b} = \begin{bmatrix} 2 \\ 8 \\ 10 \end{bmatrix}$ and our given $A = \begin{bmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{bmatrix}$

2. Permutation Matrices An $n \times n$ *permutation matrix* P has the rows of the $n \times n$ identity matrix I in any order. in other words it has exactly one 1 in each row and column.

Clearly, there are $n!$ permutation matrices of order n. (Think about how you would prove this.)

Permutation matrices have the property that $P^T = P^{-1}$.

GROUPWORK

Write down the 2! matrices of order 2 (i.e. of dimension 2×2)

Write down the 3! matrices of order 3

Exercise

Choose any one of your permutation matrices of order 3 from above and confirm that it has the property that $P^T = P^{-1}$.