Linear Systems

Math 214 Spring 2008 — Fowler 309 MWF 9:30 am - 10:25 am

 http://faculty.oxy.edu/ron/math/214/08/

Class 11: **Friday February 15**

SUMMARY Matrix Algebraic Operations
CURRENT READING Poole 3.2

Summary
Let's do math with matrices. Yay. We'll summarize our knowledge of algebraic properties of matrices.

Homework Assignment
*HW # 11: Section 3.2: 1,2,3,***4,5***,14,24,***37, 44***; EXTRA CREDIT # 45, 46 DUE WED FEB 20*

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1. Algebraic Properties of Matrix Addition and Scalar Multiplication

Let A,B and C be matrices of size $m \times n$ and let O be the zero matrix of size $m \times n$. Let c and d be scalars.

(1) $A + B = B + A$ (Commutativity of Addition)

(2) $A + O = A$ (Existence of Additive Identity)

(3) $A + (-A) = O$ (Existence of Additive Inverse)

(4) $c(A + B) = cA + cB$ (Distributivity of Scalar Multiplication)

(5) $(c + d)A = cA + dA$ (Distributivity of Scalar Addition)

(6) $(cd)A = c(dA)$ (Distributivity of Scalar Multiplication)

2. Algebraic Properties of Matrix Multiplication

(1) $A(BC) = (AB)C$ (Associativity of Matrix Multiplication)

(2) $A(B + C) = AB + AC$ (Distributivity of Left Matrix Multiplication)

(3) $(A + B)C = AC + BC$ (Distributivity of Right Matrix Multiplication)

(4) $k(AB) = (kA)B = A(kB)$ (Associativity of Scalar Multiplication)

(5) $I_m A = A = AI_n$ (Existence of Multiplicative Identity)

(6) $(cd)A = c(dA)$ (Distributivity of Scalar Multiplication)

(7) $1A = A$ (Existence of Multiplicative Identity)

Exercise
Is $(A + B)^2 = A^2 + 2AB + B^2$ for all matrices A and B? Prove your answer!

3. Linear Independence and Span With Matrices

Recall we previously defined the concepts of **linear independence** and **span** involving vectors in $\mathbb{R}^n$.

GroupWork

Write down a one sentence definition in YOUR OWN WORDS explaining linear independence and span.

Linear Independence

Span

EXAMPLE

Consider $A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Are these *matrices* linearly independent? What is the span of these matrices?

CLICKER QUESTION 11.1

If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$ what is A^T?

A. $A^T = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$

B. $A^T = \begin{bmatrix} 2 & 0 & -2 \\ 3 & -1 & 0 \\ 1 & 3 & 4 \end{bmatrix}$

C. $A^T = \begin{bmatrix} -2 & 0 & 4 \\ 0 & -1 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

D. $A^T = \begin{bmatrix} 1 & 3 & 4 \\ 3 & -1 & 0 \\ 2 & 0 & -2 \end{bmatrix}$

CLICKER QUESTION 11.2

If $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ then calculate the product AB.

A. $AB = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

B. $AB = \begin{bmatrix} 10 & 7 \end{bmatrix}$

C. $AB = \begin{bmatrix} 8 & 4 \\ -3 & -2 \end{bmatrix}$

D. $AB = \begin{bmatrix} 7 \\ 10 \end{bmatrix}$

E. None of the above.

F. This matrix multiplication is impossible.

CLICKER QUESTION 11.3

If A and B are both 2x3 matrices, then which of the following is not defined?

A. $A + B$

B. $A^T B$

C. BA

D. AB^T

E. More than one of the above

F. All of these are defined.

CLICKER QUESTION 11.4

Calculate $\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$.

A. $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

B. $\begin{bmatrix} 0 & -2 \\ 2 & 5 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 0 \\ -6 & 2 \end{bmatrix}$

D. None of the above.

E. This matrix multiplication is impossible.

CLICKER QUESTION 11.5

In order to compute the matrix product AB, what must be true about the sizes of A and B?

A. A and B must have the same number of rows.

B. A and B must have the same number of columns.

C. A must have as many rows as B has columns.

D. A must have as many columns as B has rows.

CLICKER QUESTION 11.6

If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{bmatrix}$ what is the (3,2)-entry of AB?
(You should be able to determine this without computing the entire matrix product.)

A. 1

B. 3

C. 4

D. 8