Linear Systems

Math 214 Spring 2008 | Fowler 309 MWF 9:30 am - 10:25 am

 | http://faculty.oxy.edu/ron/math/214/08/

Class 5: **Friday February 1**

SUMMARY Understanding Linear Systems of Equations
CURRENT READING Poole 2.1

OUTLINE
Today we will discover different ways of looking at linear systems and discover an interesting fact common to **all** linear systems.

Homework Assignment
HW #5: Section 1.3: 7, **14, 18, 27, 29***; EXTRA CREDIT 25: DUE MON FEB 4.*

GroupWork
Solve one of the following systems of equations.
System A.

$$\begin{aligned} 2x - y &= 1 \\ -4x + 2y &= 2 \end{aligned}$$

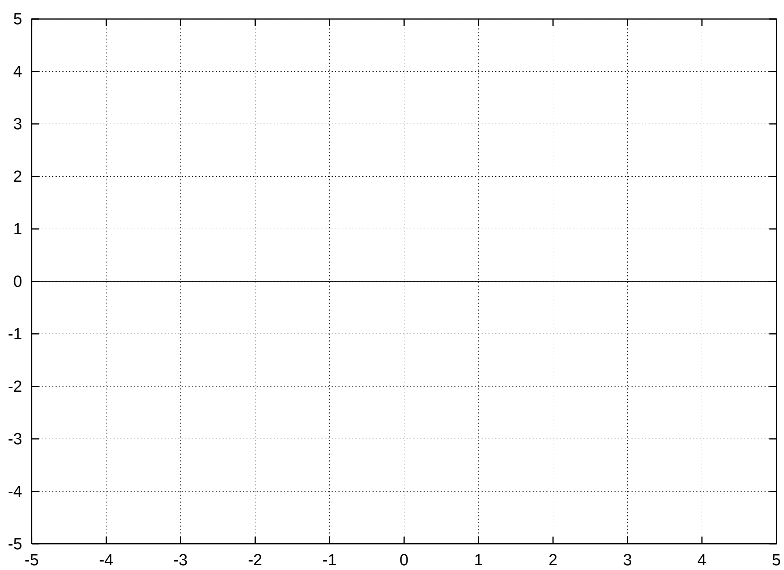
System B.

$$\begin{aligned} 2x - y &= 1 \\ -4x + y &= 2 \end{aligned}$$

System C.

$$\begin{aligned} 2x - y &= 1 \\ -6x + 3y &= -3 \end{aligned}$$

Let's graph each of the above systems of equations on the xy-plane below.
Q: Before doing so, what do you *expect* to see? What **do** you see?



1. Algebraic and Geometric Interpretations of Linear Systems

$$\begin{aligned} 4x - y &= 4 \\ 2x - 3y &= -5 \end{aligned}$$

The above is called the **row form** of the system of equations.

Geometrically, the row form can be viewed as:

$$x \begin{bmatrix} 4 \\ 2 \end{bmatrix} + y \begin{bmatrix} -1 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

The above is called the **column form** of the system of equations.

Geometrically, the column form can be viewed as:

$$\begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

The above is called the **matrix form** of the system of equations.

Geometrically, the matrix form can be viewed as:

Warm-up

Consider 2 random lines in 2-dimensional space (the regular Cartesian plane). What are the possible scenarios these two random lines can produce? Draw pictures below:

Consider 2 random **planes** in 3-dimensional space. What are the possible scenarios these 2 random planes can produce? Write them down below.

Consider 3 random planes in 3-dimensional space. What are the possible scenarios these 3 planes can produce? Write them below.

DISCUSSION

What is the connection between the above scenarios and the question of when a linear system of equations has a unique solution? Or no solution?

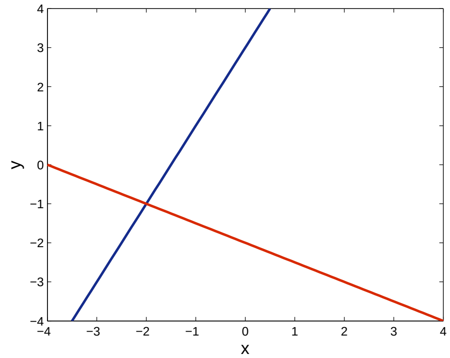
DEFINITION: **consistent**

If a system of linear equations has at least one solution then it is called a **consistent** linear system. Otherwise, it is called an **inconsistent** linear system.

DEFINITION: **singular**

If a system of linear equations does not have a unique solution then it is called a **singular** linear system. Otherwise, it is called an **non-singular** linear system.

CLICKER QUESTION 5.1



Which of the following systems of equations can be represented by the graph below?

A. $3x + 3y = -6, x + 2y = 3$

B. $x - y = -5, 2x + y = 4$

C. $-8y + 2x = 4, 2x + 4y = -8$

D. $-x + 3y = 9, 2x - y = 4$

CLICKER QUESTION 5.2

What is the solution to the following linear system of equations?

$$\begin{aligned} -3x + 2y &= 4 \\ 12x - 8y &= 10 \end{aligned}$$

A. $x = 4/3, y = 0$

B. $x = 1/2, y = -1/2$

C. $x = 0, y = 2$

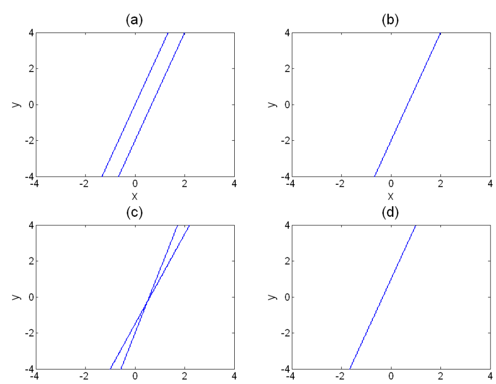
D. There are an infinite number of solutions to the system.

E. There are no solutions to the system.

CLICKER QUESTION 5.3

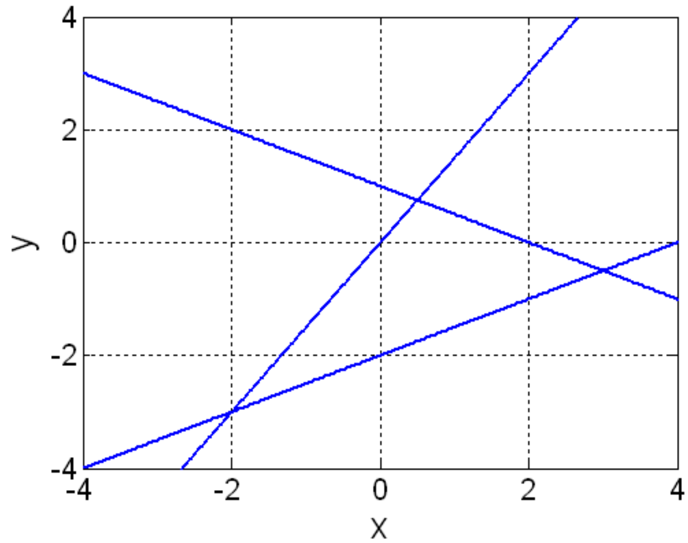
Which of the graphs below could represent the following linear system?

$$\begin{aligned} 3x - y &= 2 \\ -9x + 3y &= -6 \end{aligned}$$



CLICKER QUESTION 5.4

We have a system of three linear equations with two unknowns, as plotted in the graph shown. How many solutions does the system have?



A. 0

B. 1

C. 2

D. 3

E. Infinite.

CLICKER QUESTION 5.5

A system of linear equations could not have exactly ____________________ solutions.

A. 0

B. 1

C. 2

D. Infinite.

E. All of these are possible numbers of solutions to a system of linear equations