Linear Systems

Class 3: **Monday January 28**

SUMMARY Projections, Equations of Lines
CURRENT READING Poole 1.2 and 1.3

RECALL
At the end of the previous class we introduced the concept of a **projection** of one vector on another. We'll explore this concept a bit more and begin our foray into Section 1.3 of the text by looking at new ways to define equations of lines using vectors.

Homework Assignment #3
Section 1.2: #31, **41***, 57,* **62, 63, 64***: DUE WED JAN 31*

Projection.
For any vectors $\vec{u}$ and $\vec{v}$ where $\vec{u} \neq 0$ then **the projection of $\vec{v}$ onto $\vec{u}$** is the vector $\text{proj}_{\vec{u}}(\vec{v})$ defined by:

$$\text{proj}_{\vec{u}}(\vec{v}) = \left(\frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}}\right)\vec{u}$$

Draw a picture of the projection of $\vec{v}$ onto $\vec{u}$ in the space below:

EXAMPLE
Let's look at the derivation of this formula.

$$\begin{aligned}
\vec{p} &= |\vec{v}|\cos\theta_{uv}\hat{u} \\
&= |\vec{v}|\left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}\right)\left(\frac{\vec{u}}{|\vec{u}|}\right) \\
&= \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}|^2}\right)\vec{u} \\
&= \left(\frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}}\right)\vec{u} \\
\vec{p} &= \text{proj}_{\vec{u}}(\vec{v})
\end{aligned}$$

Poole, Page 56, #6. Find the projection of $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ onto $\vec{u} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$.

Equations of a Line in $\mathbb{R}^2$ and $\mathbb{R}^3$

The main way we often think of lines in euclidean space (i.e. the space we are used to living in where lines are perfectly "straight" and go on forever) is to define a line in $\mathbb{R}^n$ as the set of points composing the one dimensional object connecting two distinct points in space.

General Form of the Equation of a Line in $\mathbb{R}^2$

The **general form** of the equation of a line L in $\mathbb{R}^2$ is $ax + by = c$. In this case the vector $\hat{n} = \begin{bmatrix} a \\ b \end{bmatrix}$ is a **normal vector** to the line L.

Normal Form of the Equation of a Line in $\mathbb{R}^2$

The **normal form** of the equation of a line L in $\mathbb{R}^2$ is $\hat{n} \cdot (\vec{x} - \vec{p}) = 0$ or $\hat{n} \cdot \vec{x} = \hat{n} \cdot \vec{p})$. In this case the non-zero vector $\hat{n}$ is again a **normal vector** to the line L and $\vec{p}$ is a particular given point on the line L.

Vector Form of the Equation of a Line in $\mathbb{R}^2$

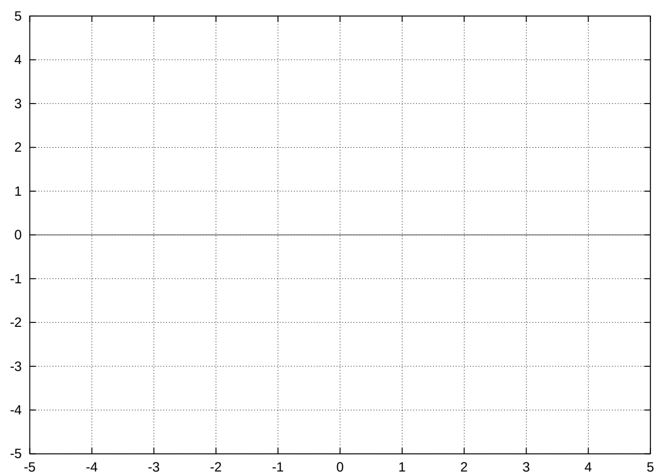
The **vector form** of the equation of a line L in $\mathbb{R}^2$ (or $\mathbb{R}^n$) is $\vec{x} = \vec{p} + t\vec{d}$. In this case the non-zero vector $\vec{d}$ is a **direction vector** for the line L and $\vec{p}$ is a particular given point on the line L.

Parametric Form of the Equation of a Line in $\mathbb{R}^2$

The **parametric form** of the equation of a line L in $\mathbb{R}^2$ (or $\mathbb{R}^n$) is the set of equations formed from the components of the **vector form** of the equation of the line. In this case those equations are $x = p_1 + d_1 t$ and $y = p_2 + d_2 t$ where $\vec{p} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$ and $\vec{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$. Note, if the vector has n components, then the parametric form of L will consist of n linear equations in the parametric variable t.

GROUPWORK

Consider the line $y = -\frac{x}{2}$. On the axes below, draw in the line as well as the direction vector $\vec{d}$, the normal vector $\hat{n}$ and an example of a point vector $\vec{p}$.



Write down the **general, vector, parametric** and **normal** forms of the equation of the line also.