# Linear Systems





*Class 1*: **Wednesday January 23**

**SUMMARY** Scalars and Vectors
**CURRENT READING** Poole 1.1 and 1.2

**INTRO**
In today's class we review the concepts of vectors and scalars. In addition, we introduce the central idea of a **linear combination** of vectors.

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*Homework Assignment #1*
*Section 1.1 # 1d, 2d, 3c, 4c, 5a, 6, 9,* **11, 15, 17, 20** *: DUE FRI JAN 26*
*EXTRA CREDIT #14*

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**1. What is a vector?**
Roughly speaking, a vector is just a "bunch of numbers"!
More precisely, a vector is an *ordered set of numbers.*

*Example* 1. [2 0] is a vector; [0 2] is also; these are two different vectors, since order matters.

$[2\ \ -5\ \ 7.1]$ is a **row vector**; $\begin{bmatrix} 2 \\ -5 \\ 7.1 \end{bmatrix}$ is a **column vector**.

**Q:** What's the difference between a row vector and a column vector?

*Note.* To save space, we sometimes write $(4, 0, -8)$ instead of $\begin{bmatrix} 4 \\ 0 \\ -8 \end{bmatrix}$. So $(4, 0, -8)$ is a column vector.

Each number in the vector is called a **component** of the vector.
**Q:** What's the second component of the vector [3 6 0]? **Ans:**

**2. Vectors can be used to represent many different things!**

*Example* 2. Start from home. Drive 6 miles East, 2 miles North. Represent this by the vector [6 2]. Then continue driving 3 miles East, 5 miles South. Represent this by $[3\ \ -5]$.
**Q:** Where are we relative to home?
**Ans:** Add the two vectors: $[6\ \ 2] + [3\ \ -5] = [9\ \ -3]$.

**NOTE** **Vectors are added component-wise: one component at a time.**

*Example* 3. I have 4 nickels, 3 dimes, and 2 quarters. You give me 3 nickels and 1 dime, and take 1 quarter. So I'm left with: $[4\ \ 3\ \ 2] + [3\ \ 1\ \ -1] = [7\ \ 4\ \ 1]$.

**A Note on Notation**
The book uses boldface letters for vectors. It is difficult to *write* in boldface. So instead we'll use "arrow notation" for vectors:
Book: Let $\mathbf{v} = [4\ \ 3]$. Let $\mathbf{w} = [5\ \ 3]$. Then $\mathbf{v} + \mathbf{w} =$?
Us: Let $\vec{v} = [4\ \ 3]$. $\vec{w} = [5\ \ 3]$. Then $\vec{v} + \vec{w} =$?

*Example* 4. $[4\ \ 2] + [3\ \ 1\ \ -1] =$?**Ans:** Undefined.

**NOTE** **Vectors of different size can NOT be added to each other.**

### 3. Multiplying a vector by a number: scalars

What's $5+5+5+5+5+5=?$
What's $[5\ \ 3]+[5\ \ 3]+[5\ \ 3]+[5\ \ 3]+[5\ \ 3]+[5\ \ 3]=?$
So, what's $6[5\ \ 3]=?$ **Ans:**

Here the number 6 is called a **scalar**. Why? Because if you draw both vectors, $[5\ \ 3]$ and $[30\ \ 18]$, on two separate $xy$-planes, they'll have different lengths but the same direction (slope): we're only changing the "scale on our map" to make one vector look like the other.

### 4. Subtracting vectors

*Example* 5. Let $\vec{v}=[4\ \ 3]$. $\vec{w}=[5\ \ 3]$. Then $\vec{v}-\vec{w}=?$ Ans: $[-1\ \ 0]$.

How can we represent vector subtraction pictorially?

Step 1. Draw $\vec{v}$.
Step 2. Multiply $\vec{w}$ by $-1$.
Step 3. Add $-\vec{w}$ to $\vec{v}$.

**Exercise** Use the space below to draw a picture of $\vec{v}-\vec{w}$.

### 5. Linear Combinations

*Example* 6. Find $a$ and $b$ such that $a[5\ \ 3]+b[3\ \ 2]=[0\ \ 1]$.

Ans: Solve two equations with two unknowns:
$5a+3b=0$
$3a+2b=1$.

We get: $a=-3$, $b=5$.

So $(-3)[5\ \ 3]+(5)[3\ \ 2]=[0\ \ 1]$. We say $[0\ \ 1]$ is a *linear combination* of $[5\ \ 3]$ and $[3\ \ 2]$.
(Books sometimes just say combination, instead of linear combination.)

DEFINITION: **linear combination**
Let $\vec{v_1},\cdots,\vec{v_n}$ be vectors. To say a vector $\vec{w}$ is a **linear combination** of $\vec{v_1},\cdots,\vec{v_n}$ means there exist scalars $c_1,\cdots,c_n\in\mathbb{R}$ such that $c_1\vec{v}_1+\cdots c_n\vec{v_n}=\vec{w}$. The numbers $c_1,\cdots,c_n$ are called **coefficients**.

*Example* 7. Is $[5\ \ 6\ \ 0]$ a linear combination of $[1\ \ 0\ \ 0]$, $[0\ \ 3\ \ 0]$, and $[0\ \ 0\ \ 8]$? **Ans:**

*Example* 8. Is $[5\ \ 6\ \ 0]$ a linear combination of $[0\ \ 1\ \ 1]$, $[0\ \ 3\ \ 0]$, and $[0\ \ 0\ \ 8]$? **Ans:**

*Example* 9. What are all possible lin combs of $[1\ \ 0]$ and $[0\ \ 1]$? **Ans:**

*Example* 10. What are all possible lin combs of $[1\ \ 1]$ and $[2\ \ 2]$? **Ans:**