Math 214 Spring 2008 **SHOW ALL YOUR WORK** Quiz Seven

1. Consider the matrix $A = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$ and its inverse, $A^{-1} = \begin{bmatrix} -3/4 & 1/2 \\ 1/2 & 0 \end{bmatrix}$.

a. *(2 points).* Find the eigenvalues, λ_1 and λ_2, of A

$p(\lambda) = \lambda^2 - tr(A)\lambda + det(A) = 0$

$\lambda^2 - 3\lambda - 4 = 0$

$(\lambda - 4)(\lambda + 1) = 0$

$\det(A - \lambda I) = \begin{vmatrix} -\lambda & 2 \\ 2 & 3-\lambda \end{vmatrix} = -\lambda(3-\lambda) - 4 = \lambda^2 - 3\lambda - 4$

$\lambda_1 = 4 \quad \lambda_2 = -1$

b. *(4 points).* Find the corresponding eigenvectors $\vec{x}_1$ and $\vec{x}_2$ of A

$\lambda_1 = 4$ Find $null(A - 4I)$

$\left(\begin{array}{cc|c} -4 & 2 & 0 \\ 2 & -1 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$

$2x - y = 0 \Rightarrow y = 2x$

$\vec{x}_1 = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 2x \end{pmatrix} = x\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, x free

$\vec{x}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\lambda_2 = -1$ Find $null(A - (-1)I)$

$\left(\begin{array}{cc|c} 1 & 2 & 0 \\ 2 & 4 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right)$

$x + 2y \Rightarrow x = -2y$

$\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2y \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$; y free

$\vec{x}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

c. *(2 points).* Find the eigenvalues, $\hat{\lambda}_1$ and $\hat{\lambda}_2$, of A^{-1}.

$\lambda^2 - tr(A^{-1})\lambda + det(A^{-1}) = 0$

$\lambda^2 - \left(-\frac{3}{4}\right)\lambda + \left(-\frac{1}{4}\right) = 0$

$\lambda^2 + \frac{3}{4}\lambda - \frac{1}{4} = 0 \quad 4\lambda^2 + 3\lambda - 1 = 0$

$\cancel{(2\lambda - 1)(2\lambda + 1)}$

$(4\lambda - 1)(\lambda + 1) = 0 \Rightarrow \lambda = \frac{1}{4}, -1$

d. *(2 points).* Confirm that the eigenvectors of A^{-1} are the same eigenvectors, $\vec{x}_1$ and $\vec{x}_2$, as A (from part b.)

$\begin{pmatrix} -3/4 & 1/2 \\ 1/2 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -3/4 + 1 \\ 1/2 + 0 \end{pmatrix} = \begin{pmatrix} 1/4 \\ 1/2 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \hat{\lambda}_1 = \frac{1}{4} \quad \hat{\vec{x}}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\begin{pmatrix} -3/4 & 1/2 \\ 1/2 & 0 \end{pmatrix}\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3/2 + 1/2 \\ -1 + 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} = -1\begin{pmatrix} -2 \\ 1 \end{pmatrix} \quad \hat{\lambda}_2 = -1, \hat{\vec{x}}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$