Math 214 Spring 2008 **SHOW ALL YOUR WORK** Quiz **Four**

A matrix of real numbers A is said to be **idempotent** if it's equal to its own square, in other words $A^2 = A$.

Consider the following matrices, identify which of them are idempotent.

EXPLAIN YOUR ANSWERS.

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ is not defined since $m \neq n$ one cannot multiply this matrix by itself

NOT IDEMPOTENT

(b) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \neq \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \Rightarrow A\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ is not idempotent, since $A^2 \neq A$

(c) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \Rightarrow A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ IS IDEMPOTENT since $A^2 = A$

(d) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \Rightarrow A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ is NOT idempotent.

(e) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$ is not square so can not be multiplied by itself, so it is NOT IDEMPOTENT.