Math 214 Spring 2008 **SHOW ALL YOUR WORK** Quiz **Three**

1. Consider the system of equations below, where a is an unknown parameter.

$$\begin{aligned} ax + 3y &= -3 \\ 4x + 6y &= 6 \end{aligned}$$

a. *(6 points).* Use elimination to form the upper-diagonal form of the augmented coefficient matrix for this system. Back substitute to get solutions for x and y in terms of the parameter a. What assumption(s) about a do you have to make to do this?

$$\left(\begin{array}{cc|c} a & 3 & -3 \\ 4 & 6 & 6 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|c} 4 & 6 & 6 \\ a & 3 & -3 \end{array}\right) \xrightarrow{R_1' = R_1 \cdot \frac{1}{2}} \left(\begin{array}{cc|c} 2 & 3 & 3 \\ a & 3 & -3 \end{array}\right)$$

$$\xrightarrow{R_2' = R_2 - \frac{a}{2}R_1} \left(\begin{array}{cc|c} 2 & 3 & 3 \\ 0 & 3-\frac{3a}{2} & -3-\frac{3a}{2} \end{array}\right) \longleftarrow \left(\begin{array}{cc|c} 2 & 3 & 3 \\ 0 & \frac{6-3a}{2} & \frac{-6-3a}{2} \end{array}\right)$$

$$\frac{6-3a}{2}\,y = \frac{-6-3a}{2} \Rightarrow y = \frac{-6-3a}{6-3a} \qquad 6-3a \neq 0,\ a \neq 2$$

$$= \frac{-2-a}{2-a} = \boxed{\frac{a+2}{a-2} = y}$$

$$2x + 3y = 3$$

Condition $a \neq 2$

$$\boxed{x = \frac{6}{2-a} = \frac{-6}{a-2} = x}$$

$$2x = 3 - 3\left(\frac{a+2}{a-2}\right)$$

$$2x = \frac{3a-6-3a-6}{a-2} = \frac{-12}{a-2}$$

b. *(2 points).* If $a = 0$ how many solutions does this system have? Either find the solution(s) or explain why the system can not be solved.

If $a = 0$, $x = \frac{6}{2-0}$, $y = \frac{0+2}{0-2} \Rightarrow x = 3, y = -1$. There's only one solution when $a = 0$.

c. *(2 points).* If $a = 2$ how many solutions does this system have? Either find the solution(s) or explain why the system can not be solved.

If $a = 2$

$$\left(\begin{array}{cc|c} 2 & 3 & -3 \\ 4 & 6 & 6 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 2 & 3 & -3 \\ 0 & 0 & 12 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 3/2 & -3/2 \\ 0 & 0 & 12 \end{array}\right)$$

NO SOLUTION.

There are no values x and y

$0x + 0y = 12$