Math 214 Spring 2008 SHOW ALL YOUR WORK Quiz One

1. Consider $\vec{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} k^2 \\ k \\ -3 \end{bmatrix}$ where k is some unknown scalar.

(a) *3 points.* Find the values of the scalar k for which the two vectors $\vec{u}$ and $\vec{v}$ are **orthogonal** to each other.

$$\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} K^2 \\ K \\ -3 \end{pmatrix} = 0$$

$$K^2 - K - 6 = 0$$

$$(K-3)(K+2) = 0 \Rightarrow K = 3 \text{ or } -2$$

(b) *2 points.* Is it possible to find values of k for which the two vectors $\vec{u}$ and $\vec{v}$ are **parallel** to each other? EXPLAIN YOUR ANSWER.

$$\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \alpha \begin{pmatrix} K^2 \\ K \\ -3 \end{pmatrix} \Rightarrow \begin{aligned} 1 &= \alpha K^2 \\ -1 &= \alpha K \\ 2 &= \alpha(-3) \Rightarrow \alpha = -\frac{2}{3} \end{aligned}$$

$$K^2 = \frac{1}{\alpha} = \frac{1}{-2/3} = -\frac{3}{2}$$ NO SOLUTION for K. There

(c) *3 points.* Let $k = 0$ to produce a specific known vector $\vec{v}$. Compute $\text{proj}_{\vec{v}}(\vec{u})$ and $\text{proj}_{\vec{u}}(\vec{v})$.

$$\vec{v} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$$

$$\text{proj}_{\vec{v}}(\vec{u}) = \frac{\begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}}{\begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}} \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} = \frac{-6}{9} \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} = -\frac{2}{3} \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

$$\text{proj}_{\vec{u}}(\vec{v}) = \frac{\begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}}{\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \frac{-6}{1+1+4} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$$

(d) *2 points.* Are your answers in part (c) different? Is this a surprise? EXPLAIN YOUR ANSWER.

The two vectors are different, as expected since projection of $\vec{v}$ in the $\vec{u}$ direction and the projection of $\vec{u}$ in the $\vec{v}$ direction, since $\vec{u}$ and $\vec{v}$ are different.