Math 214 Spring 2008 BONUS GROUP QUIZ 1

EXPLAIN YOUR ANSWERS

Group Members: BUCKMIRE

Goal: To understand the connections between reduced row echelon form of a coefficient matrix and the solutions of the corresponding linear system.

Consider the following lines in $\mathbb{R}^2$: $ax + by = e$ and $cx + dy = f$, where a, b, c, d, e and f are all real numbers such that there are two lines (even if those lines appear on top of each other).

(a) (*2 points*). Write down the augmented coefficient matrix of the linear system which represents the point of intersection of the lines.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e \\ f \end{pmatrix} \to \left(\begin{array}{cc:c} a & b & e \\ c & d & f \end{array}\right)$$

(b) (*4 points*). Considering the coefficient matrix, what are *all* the possible 2x2 matrices in reduced row echelon form that can result?

All 2x2 rrefs: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & \star \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

The rref's representing two lines intersecting are $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or $\begin{pmatrix} 1 & \star \\ 0 & 0 \end{pmatrix}$

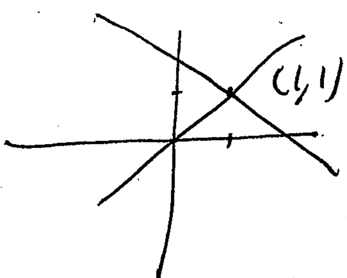
(c) (*2 points*). Write down examples of actual lines which would result in the rref matrices you found in (b) by choosing your own values for a, b, c, d, e and f.

$2x + y = 3$
$x + y = 1$ (2 non-parallel lines) $\to \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ as rref

$\left.\begin{array}{l} 2x + 2y = 2 \\ x + y = 1 \end{array}\right\}$ same line $\to \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

(d) (*2 points*). For the three possible scenarios of lines intersecting in $\mathbb{R}^2$ indicate which matrices youwrote down in (b) and (c) correspond to which geometric scenario and how many solutions the corresponding linear system has.

$\left(\begin{array}{cc:c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right)$ $\left(\begin{array}{cc:c} 1 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right)$ $\left(\begin{array}{cc:c} 1 & 1 & 1 \\ 0 & 0 & 1 \end{array}\right)$



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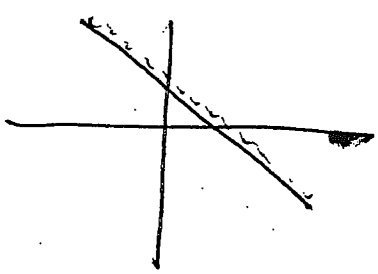

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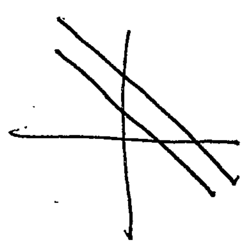

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