Math 214 Spring 2006 **SHOW ALL YOUR WORK** BONUS Quiz 10

1. *10 points.* **Poole, page 429, #6.** Let $A = \begin{bmatrix} 1/2 & a \\ b & c \end{bmatrix}$. Find all possible values of a, b and c which make A an orthogonal matrix. **How many such matrices are there?**

$A^TA = I$

$\begin{pmatrix} 1/2 & b \\ a & c \end{pmatrix}\begin{pmatrix} 1/2 & a \\ b & c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\frac{1}{4} + b^2 = 1$ $\quad \frac{a}{2} + bc = 0$

$\frac{a}{2} + bc = 0$ $\quad a^2 + c^2 = 1$

$a = -2bc$ $\quad a^2 + c^2 = 1$

$b^2 = \frac{3}{4}$

$b = \pm\frac{\sqrt{3}}{2}$

$a^2 = 4b^2c^2$

$a^2 = 3c^2$

$4c^2 = 1$

$c^2 = \frac{1}{4}$

$c = \pm\frac{1}{2}$

$b = \frac{\sqrt{3}}{2}$, $c = \frac{1}{2}$, $a = -\frac{\sqrt{3}}{2}$

$b = -\frac{\sqrt{3}}{2}$, $c = -\frac{1}{2}$, $a = -\frac{\sqrt{3}}{2}$

$b = \frac{\sqrt{3}}{2}$, $c = -\frac{1}{2}$, $a = \frac{\sqrt{3}}{2}$

$b = -\frac{\sqrt{3}}{2}$, $c = \frac{1}{2}$, $a = \frac{\sqrt{3}}{2}$

There are four matrices of this kind.

$\begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix}$ $\quad \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{pmatrix}$ $\quad \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ +\sqrt{3}/2 & -1/2 \end{pmatrix}$ $\quad \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$