Math 214 Spring 2008 **SHOW ALL YOUR WORK** **BONUS** Quiz 6

1. *10 points.* **Poole, page 362, #9.** Let $A = \begin{bmatrix} -5 & -6 & 3 \\ 3 & 4 & -3 \\ 0 & 0 & -2 \end{bmatrix}$.

a. *3 points.* Show that the characteristic polynomial is $p(\lambda) = 4 - 3\lambda^2 - \lambda^3 = (\lambda+2)^2(1-\lambda)$.

$$p(\lambda) = \det(A - \lambda I) = \begin{vmatrix} -5-\lambda & -6 & 3 \\ 3 & 4-\lambda & -3 \\ 0 & 0 & -2-\lambda \end{vmatrix} = (-2-\lambda)[(-5-\lambda)(4-\lambda)+18]$$

$$= -(\lambda+2)[-20-4\lambda+5\lambda+\lambda^2+18]$$

$$= -(\lambda+2)[\lambda^2+\lambda-2] = -(\lambda+2)[(\lambda-1)(\lambda+2)]$$

$$= (\lambda+2)^2(1-\lambda)$$

b. *1 point.* Find all the eigenvalues of A and their algebraic multiplicities.

$p(\lambda) = 0 \Rightarrow \lambda = -2, -2, 1$

a.m. of $\lambda = -2$ is 2

a.m. of $\lambda = 1$ is 1

c. *3 points.* Find bases for each of the eigenspaces of A and their geometric multiplicities.

$$E_{-2} = \text{null}(A+2I) = \left(\begin{array}{ccc|c} -3 & -6 & 3 & 0 \\ 3 & 6 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) = \text{span}\left\{\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right\}$$

$$E_1 = \text{null}(A-I) = \left(\begin{array}{ccc|c} -6 & -6 & 3 & 0 \\ 3 & 3 & -3 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \to \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \text{span}\left\{\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}\right\}$$

d. *3 points.* Determine whether A is diagonalizable. If A is not diagonalizable, explain why not. If A is diagonalizable, find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$ and $A = PDP^{-1}$.

Since a.m. = g.m A is diagonalizable.

$$P = \begin{pmatrix} -2 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad D = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$AP = PD$

$$\begin{pmatrix} -5 & -6 & 3 \\ 3 & 4 & -3 \\ 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} -2 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{matrix} 4 & -2 & -1 \\ -2 & 0 & 1 \\ 0 & -2 & 0 \end{matrix}$$

equal

$$\begin{pmatrix} -2 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{matrix} 4 & -2 & -1 \\ -2 & 0 & 1 \\ 0 & -2 & 0 \end{matrix}$$