Math 214 Spring 2008 **BONUS QUIZ 5**

EXPLAIN YOUR ANSWERS

1. **TRUE or FALSE** – put your answer in the box. That answer is worth 1 point. To receive ANY credit, you must also give a brief, and correct, explanation in support of your answer! Remember a statement is TRUE only if it is ALWAYS true, and it is FALSE if there exists an example which makes it FALSE.

(a) (*3 points*) **TRUE or FALSE**: "For any $n \times n$ matrix A there exists a real number λ and a $n \times 1$ vector $\vec{x}$ such that $A\vec{x} = \lambda\vec{x}$."

$\boxed{\text{FALSE}}$ λ can be COMPLEX for a REAL matrix.

$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ has characteristic polynomial $\lambda^2 + 1 = 0 \Rightarrow \lambda = \pm i$

(b) (*3 points*) **TRUE or FALSE**: "A is singular (not invertible) if and only if A has at least one zero eigenvalue." In other words, IF A is singular, THEN A has at least one zero eigenvalue *AND* IF A has at least one zero eigenvalue, THEN A is singular.

$\boxed{\text{TRUE}}$

A singular $\Longrightarrow$ A has at least one zero eigenvalue

A singular $\Leftrightarrow$ $\det(A) = 0$ but $\det(A) = \lambda_1 \lambda_2 \cdots \lambda_n = 0$ so one of these $\lambda_i = 0$

A has at least one zero eigenvalue $\Rightarrow$ A is singular

$\lambda_i = 0 \Rightarrow \prod_{i=1}^{n} \lambda_i = 0 \Rightarrow \det(A) = 0 \Rightarrow$ A is singular

Could also prove CONTRAPOSITIVE:

A ~~has~~ is INVERTIBLE $\Leftrightarrow$ A does not have a zero eigenvalue

(c) (*4 points*) **TRUE or FALSE**: "The eigenvectors of A^T are the same as the eigenvectors of A."

$\boxed{\text{FALSE}}$ The eigenvalues of A^T and A are the same.

The eigenvectors of A^T and A are not the same.

Just choose A NOT symmetric

$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ $\lambda^2 - 2\lambda + 1 = 0$, $(\lambda - 1)^2 = 0$, $\lambda = 1, 1$

$A^T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $\lambda^2 - 2\lambda + 1 = 0$, $\lambda = 1, 1$

$E_1 = \text{span}\left\{\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\} = \text{null}(A - I) = \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right)$

$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} 1$

$\text{null}(A^T - I) = \left(\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $E_1 = \text{span}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ✓