Math 214 Spring 2008 GROUP QUIZ 2

EXPLAIN YOUR ANSWERS

Goal: To synthesize understanding of the concepts of associated subspaces of a matrix, rank theorem, span, basis and linear independence.

$$\text{Given } A = \begin{bmatrix} 1 & 5 & 3 & 1 & 0 \\ -1 & -3 & 0 & 0 & 2 \\ 3 & -3 & 1 & -6 & 1 \\ 2 & -4 & -1 & -5 & 0 \end{bmatrix} \text{ with } \operatorname{rref}(A) = R = \begin{bmatrix} 1 & 0 & 0 & -1.5 & -0.5 \\ 0 & 1 & 0 & 0.5 & -0.5 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Fill in the blanks. Write in explanations in the gaps between questions.

a. col(A) is a subspace of $\mathbb{R}^4$.

col(A) = set of all possible linear combinations of column vectors of A. There are 4 component vectors, so col(A) $\subset \mathbb{R}^4$

b. The rank of the matrix A is 3.

rank is # of non-zero rows in rref(A).

c. null(A) is a subspace of $\mathbb{R}^5$.

null(A) = set of all solutions to $A\vec{x} = \vec{0}$. Since A is 4x5, these $\vec{x}$ must be 5x1 vectors. Thus null(A) $\subset \mathbb{R}^5$

d. The dimension of col(A) is 3.

rank(A) = dim(col(A)) = dim(row(A))

e. There are 3 vectors in a basis of row(A).

The number of vectors in a basis for row(A) = dimension of row(A) = rank(A) = 3

f. row(A) is a subspace of $\mathbb{R}^5$.

row(A) = set of all possible linear combinations of the row vectors of A. Since the rows have 5 components, row A $\subset \mathbb{R}^5$.

g. null(A) is spanned by the vectors $\left\{ \begin{pmatrix} 1.5 \\ -0.5 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.5 \\ 0.5 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\}$

Use magic method or solve using free vars

$x_1 - 1.5x_4 - 0.5x_5 = 0$

$x_2 + 0.5x_4 - 0.5x_5 = 0$

$x_3 + \quad x_5 = 0$

x_4 & x_5 free vars

h. The span of the columns of R is all of $\mathbb{R}^3$ TRUE or FALSE (circle one).

Span is a 3-D subspace of $\mathbb{R}^5$. This is NOT the same thing as $\mathbb{R}^3$.

$$x_4 \begin{pmatrix} 1.5 \\ -0.5 \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} 0.5 \\ 0.5 \\ -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.5x_4 + 0.5x_5 \\ -0.5x_4 + 0.5x_5 \\ -x_5 \\ x_4 \\ x_5 \end{pmatrix}$$

i. $A\vec{x} = \vec{b}$ will be solvable for any $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ 0 \end{bmatrix}$. TRUE or FALSE (circle one).

$A\vec{x} = \vec{b}$ if $\vec{b}$ is in the col(A). The column space of A does not have the form $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ 0 \end{pmatrix}$, the column space of $\underline{R}$ does. They are $\underline{\text{NOT}}$ the same.

j. An example of a basis for col(A) is $\left\{ \begin{pmatrix} 1 \\ -1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 5 \\ -3 \\ -3 \\ -4 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \\ -1 \end{pmatrix} \right\}$.

Pick 3 columns of A that look like I in rref(A).