

1. TRUE or FALSE – put your answer in the box (1 point). To receive FULL credit, you must also give a brief, and correct, explanation in support of your answer! Remember if you think a statement is TRUE you must prove it is ALWAYS true. If you think a statement is FALSE then all you have to do is show there exists a counterexample which proves the statement is FALSE at least once.

(a) TRUE or FALSE? “If a set of vectors in \mathbb{R}^n is linearly dependent, then the set must contain more vectors than there are components in each vector.”

FALSE

As long as you have 2 vectors you could have a linearly dependent set.
For example, $\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \end{pmatrix} \right\}$

(b) TRUE or FALSE? “The matrix products AA^T and $A^T A$ are defined for every matrix A .”

TRUE

Let A be an $m \times n$ matrix.
 AA^T is an $m \times n \times n \times m$, i.e. $m \times m$ matrix.
 $A^T A$ is an $n \times m \times m \times n$ i.e. $n \times n$ matrix

(c) TRUE or FALSE? “There exists no idempotent matrix which is invertible.”

FALSE

I is idempotent and invertible.
 $I^2 = I$ and $I^{-1} = I$.