Math 214 Spring 2008 **SHOW ALL YOUR WORK** **BONUS** Quiz 2

**1.** *6 points.* Consider the linear system

$$\begin{aligned} 4x - 2y + z &= a \\ x + y + z &= b \end{aligned}$$

where $a$ and $b$ are real numbers. Our goal is to discover a relationship between the solution sets of this system for various values of $a$ and $b$.

**a.** *2 points.* Consider the case $a = b = 0$. This is known as the **homogeneous** case. Use Gaussian Elimination to solve the system.

$$\left(\begin{array}{ccc|c} 4 & -2 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 4 & -2 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -6 & -3 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1/2 & 0 \end{array}\right)$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1/2 & 0 \\ 0 & 1 & 1/2 & 0 \end{array}\right)$$

$$x + \tfrac{1}{2} z = 0 \qquad y + \tfrac{1}{2} z = 0 \qquad z \text{ free}$$

$$\vec{x} = \begin{pmatrix} -1/2\, z \\ -1/2\, z \\ z \end{pmatrix} = z \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$$

**b.** *2 points.* What is the geometric interpretation or "shape" of the solution? Is it a point in $\mathbb{R}^2$? A point in $\mathbb{R}^3$? A line in $\mathbb{R}^2$? A line in $\mathbb{R}^3$? A plane in $\mathbb{R}^3$? Something else?

$\vec{x} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} t,\ t \in \mathbb{R}$ is a LINE in $\mathbb{R}^3$

**c.** *2 points.* Express your solution in vector form, i.e. $\vec{x} = \vec{p} + t\vec{d}$.

$$\vec{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$$

**2.** *4 points.* Choose any non-zero value of $a$ and $b$ that you like. This is known as the **non-homogeneous** case.

**a.** *2 points.* Repeat **Question 1** (i.e. Use Gaussian Elimination to solve the system with your chosen values of $a$ and $b$) and express your answers in vector form, i.e. $\vec{x} = \vec{p} + t\vec{d}$.

$a = 1,\ b = -1$

$$\left(\begin{array}{ccc|c} 4 & -2 & 1 & 1 \\ 1 & 1 & 1 & -1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 4 & -2 & 1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 0 & -6 & -3 & 5 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 0 & 1 & 1/2 & -5/6 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1/2 & -1/6 \\ 0 & 1 & 1/2 & -5/6 \end{array}\right)$$

$$x + \tfrac{1}{2} z = -\tfrac{1}{6} \qquad y + \tfrac{1}{2} z = -5/6$$

$$x = -1/6 - 1/2\, z \qquad y = -5/6 - 1/2\, z$$

**b.** *2 points.* What is the (geometric) relationship between your solutions in **1(c)** and **2(a)**? In other words, how are the solutions to the homogeneous linear system and non-homogeneous linear system related? EXPLAIN YOUR ANSWER.

$$\vec{x} = \begin{pmatrix} -1/6 - \frac{1}{2} z \\ -5/6 - \frac{1}{2} z \\ z \end{pmatrix} = \begin{pmatrix} -1/6 \\ -5/6 \\ 0 \end{pmatrix} + z \begin{pmatrix} -1/2 \\ -1/2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1/6 \\ -5/6 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} t$$

A line in the same direction, simply shifted from the [illegible] to a point [illegible]