Math 214 [redacted] **SHOW ALL YOUR WORK** BONUS Quiz 1

1. (a) Show that the plane given by $4x - y - z = 6$ and the line given by $x = t, y = 1+2t, z = 2+3t$ intersect.

Substitute $x=t,\ y=1+2t,\ z=2+3t$ into $4x-y-z=6$

$$4(t)+(1+2t)-(2+3t)=6$$
$$4t-1-2t-2-3t=6$$
$$-3-t=6$$
$$-t=9$$
$$t=-9$$

when $t=9$, $x=-9$
$y=1+2(-9)=-17$
$z=2+3(-9)=-25$

$\begin{pmatrix}-9\\-17\\-25\end{pmatrix}$ is on the line

$$4(-9)-(-17)-(-25)\stackrel{?}{=}6$$
$$-36+17+25\stackrel{?}{=}6$$
$$-36+42\stackrel{?}{=}$$
$$6\stackrel{?}{=}6\ \checkmark$$

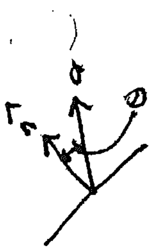
$\begin{pmatrix}-9\\-17\\-25\end{pmatrix}$ is also on the plane

(b) Find the acute angle of intersection between the line and the plane given in part (a).

Given $4x-y-z=6$, the vector $\begin{pmatrix}4\\-1\\-1\end{pmatrix}$ is orthogonal (or normal) to the plane

The line $\begin{pmatrix}t\\1+2t\\2+3t\end{pmatrix} = t\begin{pmatrix}1\\2\\3\end{pmatrix}+\begin{pmatrix}0\\1\\2\end{pmatrix}$, thus $\begin{pmatrix}1\\2\\3\end{pmatrix}$ is the direction the line is in.

The angle between the line and plane is given by the dot product formula



$$\cos\theta = \frac{(4,-1,-1)\cdot(1,2,3)}{|(4,-1,-1)||(1,2,3)|} = \frac{4-2-3}{\sqrt{4^2+1^2+1^2}\sqrt{1^2+2^2+3^2}} = \frac{-1}{\sqrt{18}\sqrt{14}}$$

$= \frac{-1}{[illegible]} = \frac{-1}{6\sqrt{7}}$, $\theta = 93.6°$ $= 1.63$ rad Actual Angle is $|90° - \theta| = 3.6°$