HW 9
11, 12, 40, 41 47EC

We let x_1, x_2, and x_3 be the number of house, special, and gourmet blends respectively. Then from the consumption of beans in each blends we get

$$\begin{aligned} 300x_1 + 200x_2 + 100x_3 &= 30,000 \\ 50x_1 + 200x_2 + 350x_3 &= 15,000 \\ 150x_1 + 100x_2 + 50x_3 &= 15,000 \end{aligned}$$

Form matrix and reduce it: $\left[\begin{array}{ccc|c} 300 & 200 & 100 & 30,000 \\ 50 & 200 & 350 & 15,000 \\ 150 & 100 & 50 & 15,000 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 60 \\ 0 & 1 & 2 & 60 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow$

$x_1 = 60 + t$, $x_2 = 60 - 2t$, $x_3 = t$ with $t \geq -60$, $t \leq 30$, $t \geq 0 \Rightarrow 0 \leq t \leq 30$.
But we also need to maximize the profit $P \Rightarrow$
$\frac{1}{2}x_1 + \frac{3}{2}x_2 + 2x_3 = P \Rightarrow \frac{1}{2}(60 + t) + \frac{3}{2}(60 - 2t) + 2t = P \Rightarrow 120 - \frac{1}{2}t = P$.
Since $0 \leq t \leq 30$, the profit P is maximized if $t = 0$, in which case $x_1 = x_2 = 60$ and $x_3 = t$.
Therefore the merchant should make 60 house and special blends, and no gourmet blends.

The maximum profit is \$120.

7. Let x, y, z, and w be the number of FeS_2, O_2, Fe_2O_3, and SO_2 molecules respectively. Then, compare the number of iron, sulfur, and oxygen atoms in reactants and products:

$$\begin{aligned} \text{Iron}: \quad x &= 2z \\ \text{Sulfur}: \quad 2x &= w \\ \text{Oxygen}: \quad 2y &= 3z + 2w \end{aligned} \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -\frac{11}{8} & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & 0 \end{array}\right].$$

Thus $z = \frac{1}{4}w$, $y = \frac{11}{8}w$, and $x = \frac{1}{2}w$.
The smallest positive value of w that will produce integer values for all four variables is the least common denominator of $\frac{1}{2}, \frac{11}{8}, \frac{1}{2} \Rightarrow w = 8$, $x = 4$, $y = 11$, and $z = 2$.
Therefore, the balanced chemical equation is $4FeS_2 + 11O_2 \longrightarrow 2Fe_2O_3 + 8SO_2$.

8. $6CO_2 + 6H_2O \longrightarrow C_6H_{12}O_6 + 6O_2$

9. $2C_4H_{10} + 13O_2 \longrightarrow 8CO_2 + 10H_2O$

10. $2C_7H_6O_2 + 15O_2 \longrightarrow 6H_2O + 14CO_2$

11. $2C_5H_{11}OH + 15O_2 \longrightarrow 12H_2O + 10CO_2$

12. $12HClO_4 + P_4O_{10} \longrightarrow 4H_3PO_4 + 6Cl_2O_7$

13. $Na_2CO_3 + 4C + N_2 \longrightarrow 2NaCN + 3CO$

14. $2C_2H_2Cl_4 + Ca(OH)_2 \longrightarrow 2C_2HCl_3 + CaCl_2 + 2H_2O$

15. (a) By applying

We form the

Letting $f_3 =$

(b) In this case

(c) Each flow m
Thus $10 \leq t$

(d) A negative fl
For example
So, if $f_2 < 0$.

16. (a) By applying

f_1

We form the

Letting $f_4 =$

(b) If $f_4 = t = 1$

(c) Each flow m
$t \geq 5$, $t \leq 2$
Combining
we see that

(d) Reversing al
to multiplyi
operation sc
solutions wi

30. Over $\mathbb{Z}_3$, we need to go from s to t.

$$\mathsf{s} = \begin{bmatrix} 2\\2\\0\\1\\1\\0\\2\\0\\1 \end{bmatrix}, \mathsf{t} = \begin{bmatrix} 2\\2\\2\\2\\2\\2\\2\\2\\2 \end{bmatrix} \Rightarrow \left[\begin{array}{ccccccccc|c} 1&1&0&1&0&0&0&0&0&0\\ 1&1&1&0&1&0&0&0&0&0\\ 0&1&1&0&0&1&0&0&0&2\\ 1&0&0&1&1&0&1&0&0&1\\ 1&0&1&0&1&0&1&0&1&1\\ 0&0&1&0&1&1&0&0&1&2\\ 0&0&0&1&0&0&1&1&0&0\\ 0&0&0&0&1&0&1&1&1&2\\ 0&0&0&0&0&1&0&1&1&1 \end{array}\right] \longrightarrow \left[\begin{array}{ccccccccc|c} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&1\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&2\\ 0&0&0&0&1&0&0&0&0&2\\ 0&0&0&0&0&1&0&0&0&1\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&1\\ 0&0&0&0&0&0&0&0&1&2 \end{array}\right]$$

showing that a solution exists.

31. Let Grace's and Hans' ages be g and h. Then we have $g = 3h$ and $g + 5 = 2(h+5) \Leftrightarrow g = 2(h+5) - 5$, so $3h = 2h + 5 \Leftrightarrow h = 5$ and $g = 15$. So, Hans is 5 and Grace is 15.

32. Let the ages be a, b, and c. Then we have $a + b + c = 60$, $a - b = b - c$, $a + (a - b) = 3c$. We can rewrite these as

$$\begin{aligned} a + b + c &= 60 \\ a - 2b + c &= 0 \\ 2a - b - 3c &= 0 \end{aligned} \quad \text{So, row reduction} \Rightarrow \left[\begin{array}{ccc|c} 1&1&1&60\\ 1&-2&1&0\\ 2&-1&-3&0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1&0&0&28\\ 0&1&0&20\\ 0&0&1&12 \end{array}\right].$$

Thus, Annie is 28, Bert is 20, and Chris is 12.

33. Let the areas be a and b. We have the following equations:

$$\begin{aligned} a + b &= 1800 \\ \frac{2}{3}a + \frac{1}{2}b &= 1100 \end{aligned}$$

We solve these to find that $a = 1200$ square yards and $b = 600$ square yards.

34. Let x_1, x_2, x_3 be the number of bundles of the first, second, and third types of corn. Then from the number of bundles in each given measure we get the following system:

$$\begin{aligned} 3x_1 + 2x_2 + x_3 &= 39 \\ 2x_1 + 3x_2 + x_3 &= 34 \\ x_1 + 2x_2 + 3x_2 &= 26 \end{aligned}$$

We form the augmented matrix and row reduce it into reduced row echelon form:

$$\left[\begin{array}{ccc|c} 3&2&1&39\\ 2&3&1&34\\ 1&2&3&26 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1&0&0&\frac{37}{4}\\ 0&1&0&\frac{17}{4}\\ 0&0&1&\frac{11}{4} \end{array}\right]$$

Therefore there are 9.25 measures of corn in a bundle of the first type, 4.25 measures of corn in a bundle of the second type, and 2.75 measures of corn in a bundle of the third type.

40. (a) We know that the three points $(0,1)$, $(-1,4)$, and $(2,1)$ must satisfy the equation $x^2+y^2+ax+by+c=0$. Plugging in these points, we get

$$\begin{aligned} 1+b+c &= 0 \\ 1+16-a+4b+c &= 0 \\ 4+1+2a+b+c &= 0 \end{aligned}$$

We form the augmented matrix and perform Gauss-Jordan elimination to get

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & -1 \\ 1 & -4 & -1 & 17 \\ 2 & 1 & 1 & -5 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

Thus $a=-2$, $b=-6$, $c=5$, and the equation of the circle is $x^2+y^2-2x-6y+5=0$. By completing the square and simplifying we get the equation $(x-1)^2+(y-3)^2=5$. Thus the center of this circle is at $(1,3)$ and the radius is $r=\sqrt{5}$.

(b) We know that the three points $(-3,1)$, $(-2,2)$, and $(-1,5)$ must satisfy the equation $x^2+y^2+ax+by+c=0$. By plugging in these points we get the system of equations

$$\begin{aligned} 9+1-3a+b+c &= 0 \\ 4+4-2a+2b+c &= 0 \\ 1+25-a+5b+c &= 0 \end{aligned}$$

We form the augmented matrix and perform Gauss-Jordan elimination to get

$$\left[\begin{array}{ccc|c} 3 & -1 & -1 & 10 \\ 2 & -2 & -1 & 8 \\ 1 & -5 & -1 & 26 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 36 \end{array}\right]$$

$\Rightarrow a=12$, $b=-10$,and $c=36 \Rightarrow$ the equation is $x^2+y^2+12x-10y+36=0$. By completing the square and simplifying we get the equation $(x+6)^2+(y-5)^2=25$. The center of this circle is at $(-6,5)$ and the radius is $r=5$.

41. We have: $\frac{3x+1}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3} \Leftrightarrow (x+3)\,A + (x-1)\,B = 3x+1 \Leftrightarrow$
$x\,(A+B) + (3A-B) = 3x+1$.
Equating the coefficients of x and constants we get:
$A+B=3,\ 3A-B=1 \Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 3 & -1 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right] \Rightarrow A=1,\ B=2 \Rightarrow$
The partial fraction decomposition is $\frac{3x+1}{x^2+2x-3} = \frac{1}{x-1} + \frac{2}{x+3}$.

Assume $1^2 + 2^2 + \cdots + n^2 = an^3 + bn^2 + cn + d$, and let $n = 0, 1, 2, 3$ to get

$$\begin{aligned} d &= 0 \\ a+b+c+d &= 1 \\ 8a+4b+2c+d &= 5 \\ 27a+9b+3c+d &= 14 \end{aligned}$$

From these we form the augmented matrix and perform Gauss-Jordan elimination to get

$$\left[\begin{array}{cccc|c} 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 & 5 \\ 27 & 9 & 3 & 1 & 14 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

Thus $a = \frac{1}{3}$, $b = \frac{1}{2}$, $c = \frac{1}{6}$, $d = 0$, and we find that

$$1^2 + 2^2 + \cdots + n^2 = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n = \frac{n(n+1)(2n+1)}{6}.$$

47. Assume $1^3 + 2^3 + \cdots + n^3 = an^4 + bn^3 + cn^2 + dn + e$, and let $n = 0, 1, 2, 3, 4$ to get

$$\begin{aligned} e &= 0 \\ a+b+c+d+e &= 1 \\ 16a+8b+4c+2d+e &= 9 \\ 81a+27b+9c+3d+e &= 36 \\ 256a+64b+16c+4d+e &= 100 \end{aligned}$$

From these we form the augmented matrix and perform Gauss-Jordan elimination to get

$$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 16 & 8 & 4 & 2 & 1 & 9 \\ 81 & 27 & 9 & 3 & 1 & 36 \\ 256 & 64 & 16 & 4 & 1 & 100 \end{array}\right] \longrightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

Thus $a = \frac{1}{4}$, $b = \frac{1}{2}$, $c = \frac{1}{4}$, $d = 0$, $e = 0$, and we find that

$$1^3 + 2^3 + \cdots + n^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2 = \frac{1}{4}n^2(n+1)^2 = \left(\frac{n(n+1)}{2}\right)^2.$$