HW8

2.3 Spanning Sets and Linear Independence

1. As in Example 2.18, we want to find scalars x and y such that:

$$x\begin{bmatrix}1\\-1\end{bmatrix}+y\begin{bmatrix}2\\-1\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$$ Expanding, we obtain the system: $$\begin{aligned}x+2y&=1\\-x-y&=2\end{aligned}$$

We then row reduce the associated augmented matrix: $\left[\begin{array}{rr|r}1&2&1\\-1&-1&2\end{array}\right]\longrightarrow\left[\begin{array}{rr|r}1&0&-5\\0&1&3\end{array}\right]$

So the solution is $x=-5$, $y=2$, and the linear combination is $-5\begin{bmatrix}1\\-1\end{bmatrix}+3\begin{bmatrix}2\\-1\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$.

2. As in Example 2.18, we want to find scalars x and y such that:

$$x\begin{bmatrix}4\\-2\end{bmatrix}+y\begin{bmatrix}-2\\1\end{bmatrix}=\begin{bmatrix}2\\1\end{bmatrix}$$ Expanding, we obtain the system: $$\begin{aligned}4x-2y&=2\\-2x+y&=1\end{aligned}$$

We then row reduce the associated augmented matrix: $\left[\begin{array}{rr|r}4&-2&2\\-2&1&1\end{array}\right]\longrightarrow\left[\begin{array}{rr|r}4&-2&2\\0&0&2\end{array}\right]$

Since $0\neq 2$, this system clearly has no solution. So, what do we conclude?
We conclude that $\mathbf{v}$ is not a linear combination of $\mathbf{u}_1$ and $\mathbf{u}_2$.

We could have noted $\begin{bmatrix}4\\-2\end{bmatrix}=-2\begin{bmatrix}-2\\1\end{bmatrix}$, while $\begin{bmatrix}2\\1\end{bmatrix}$ is not a multiple of $\begin{bmatrix}-2\\1\end{bmatrix}$.

3. As in Example 2.18, we want to find scalars x and y such that:

$$x\begin{bmatrix}1\\1\\0\end{bmatrix}+y\begin{bmatrix}0\\1\\1\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$ Expanding, we obtain the system: $$\begin{aligned}x&=1\\x+y&=2\\y&=3\end{aligned}$$

Since $x=1$ and $y=3$ implies $x+y\neq 2$, this system clearly has no solution.
Therefore, $\mathbf{v}$ is not a linear combination of $\mathbf{u}_1$ and $\mathbf{u}_2$.

4. As in Example 2.18, we want to find scalars x and y such that:

$$x\begin{bmatrix}1\\1\\0\end{bmatrix}+y\begin{bmatrix}0\\1\\1\end{bmatrix}=\begin{bmatrix}3\\2\\-1\end{bmatrix}$$ Expanding, we obtain the system: $$\begin{aligned}x&=3\\x+y&=2\\y&=-1\end{aligned}$$

Since $x=3$ and $y=-1$ implies $x+y=2$, those values of x and y are clearly the solution.

So the solution is $x=3$, $y=-1$, and the linear combination is $3\begin{bmatrix}1\\1\\0\end{bmatrix}-1\begin{bmatrix}0\\1\\1\end{bmatrix}=\begin{bmatrix}3\\2\\-1\end{bmatrix}$.

2. Similar to Example 2.19, show $x\begin{bmatrix}1\\2\\3\end{bmatrix}+y\begin{bmatrix}-1\\-1\\0\end{bmatrix}+z\begin{bmatrix}2\\1\\-1\end{bmatrix}=\begin{bmatrix}a\\b\\c\end{bmatrix}$ can always be solved.

The augmented matrix is $\left[\begin{array}{ccc|c}1&-1&2&a\\2&-1&1&b\\3&0&-1&c\end{array}\right]$, and row reduction produces:

$$\left[\begin{array}{ccc|c}1&-1&2&a\\2&-1&1&b\\3&0&-1&c\end{array}\right]\xrightarrow[R_3-3R_1]{R_2-2R_1}\left[\begin{array}{ccc|c}1&-1&2&a\\0&1&-3&-2a+b\\0&3&-7&-3a+c\end{array}\right]\xrightarrow[R_3-3R_2]{R_1+R_2}\left[\begin{array}{ccc|c}1&0&-1&-a+b\\0&1&-3&-2a+b\\0&0&2&3a-3b+c\end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_3}\left[\begin{array}{ccc|c}1&0&-1&-a+b\\0&1&-3&-2a+b\\0&0&1&(3a-3b+c)/2\end{array}\right]\xrightarrow[R_2+3R_3]{R_1+R_3}\left[\begin{array}{ccc|c}1&0&0&(a-b+c)/2\\0&1&0&(5a-7b+3c)/2\\0&0&1&(3a-3b+c)/2\end{array}\right]$$

We see that $x=(a-b+c)/2$, $y=(5a-7b+3c)/2$, and $z=(3a-3b+c)/2$.
So for any choice of a, b, and c we have:

$$\left(\frac{a-b+c}{2}\right)\begin{bmatrix}1\\2\\3\end{bmatrix}+\left(\frac{5a-7b+3c}{2}\right)\begin{bmatrix}-1\\-1\\0\end{bmatrix}+\left(\frac{3a-3b+c}{2}\right)\begin{bmatrix}2\\1\\-1\end{bmatrix}=\begin{bmatrix}a\\b\\c\end{bmatrix}$$

13. We should describe the span of the given vectors (*a*) geometrically and (*b*) algebraically.

(a) Geometrically, we can see that the set of all linear combinations of $\begin{bmatrix}2\\-4\end{bmatrix}$ and $\begin{bmatrix}-1\\2\end{bmatrix}$ is just the line through the origin with $\begin{bmatrix}-1\\2\end{bmatrix}$ as direction vector.

Why do we not have to consider $\begin{bmatrix}2\\-4\end{bmatrix}$? Because $\begin{bmatrix}2\\-4\end{bmatrix}=-2\begin{bmatrix}-1\\2\end{bmatrix}$.

(b) Algebraically, the vector equation of this line is $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}t$.

That is just another way of saying that $\begin{bmatrix}x\\y\end{bmatrix}$ is in the span of $\begin{bmatrix}-1\\2\end{bmatrix}$.

Suppose we want to obtain the general equation of this line.
One method is to use the system of equations arising from the vector equation:

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}t\Rightarrow\begin{array}{rcl}x&=&-t\\y&=&2t\end{array}\quad\text{So } y=2(-x)=-2x\Rightarrow 2x+y=0.$$

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14. We should describe the span of the given vectors (*a*) geometrically and (*b*) algebraically.

(a) Geometrically, we can see that the set of all linear combinations of $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ is just the line through the origin with $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ as direction vector.

Why do we not have to consider $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$? Because $\begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0\begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

(b) Algebraically, the vector equation of this line is $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} t$.

That is just another way of saying that $\begin{bmatrix} x \\ y \end{bmatrix}$ is in the span of $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$.
Suppose we want to obtain the general equation of this line.
One method is to use the system of equations arising from the vector equation:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} t \Rightarrow \begin{aligned} x &= 3t \\ y &= 4t \end{aligned} \quad \text{So } y = 4\left(\tfrac{1}{3}x\right) \Rightarrow 3y = 4x \Rightarrow 4x - 3y = 0.$$

15. We should describe the span of the given vectors (*a*) geometrically and (*b*) algebraically.

(a) Geometrically, we can see that the set of all linear combinations of $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$ is just the plane through the origin with $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$ as direction vectors.

(b) Algebraically, the vector equation of this plane is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = s\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + t\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$.

That is just another way of saying that $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is in the span of $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$.
Suppose we want to obtain the general equation of this plane.
One method is to use the system of equations arising from the vector equation:

$$\begin{aligned} s + 3t &= x \\ 2s + 2t &= y \\ -t &= z \end{aligned} \Rightarrow \left[\begin{array}{cc|c} 1 & 3 & x \\ 2 & 2 & y \\ 0 & -1 & z \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 3 & x \\ 0 & -4 & -2x+y \\ 0 & 0 & (2x-y+4z)/4 \end{array}\right]$$

We know this system is consistent, since $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ *is* in the span of $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$.

So, we *must* have $2x - y + 4z = 0$, giving us the general equation we seek.

Note: Both $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$ are orthogonal to $\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$. Should they be?

22. The vectors $\mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}$ are linearly independent.

This can be determined by inspection because they are not scalar multiples of each other.

23. Since there is no obvious dependence relation here, we follow Example 2.23.

Find scalars c_1, c_2, and c_3 such that: $c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

Form the linear system, its associated augmented matrix, and row reduce to solve:

$$\begin{aligned} c_1 + c_2 + c_3 &= 0 \\ c_1 + 2c_2 - c_3 &= 0 \\ c_1 + 3c_2 + 2c_3 &= 0 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 2 & -1 & 0 \\ 1 & 3 & 2 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Since $c_1 = c_2 = c_3 = 0$ is the unique solution, the vectors are linearly independent.

24. Since there is no obvious dependence relation here, we follow Example 2.23.

Find scalars c_1, c_2, and c_3 such that: $c_1 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

Form the linear system, its associated augmented matrix, and row reduce to solve:

$$\begin{aligned} 2c_1 + 3c_2 + c_3 &= 0 \\ 2c_1 + c_2 - 5c_3 &= 0 \\ c_1 + 2c_2 + 2c_3 &= 0 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 2 & 3 & 1 & 0 \\ 2 & 1 & -5 & 0 \\ 1 & 2 & 2 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Since $c_1 = 4c_3$ and $c_2 = -3c_3$ is a solution, the vectors are linearly dependent.

One dependence relationship is: $4 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} - 3 \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Are there others?

25. The vectors $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ are linearly dependent.

This can be determined by inspection because $\mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3 = \mathbf{0}$.

he vectors $\mathbf{v}_1 = \begin{bmatrix} -2 \\ 3 \\ 7 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$, and $\mathbf{v}_4 = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$ are linearly dependent.

'his can be determined by inspection because $[A|\mathbf{0}]$ obviously has a nontrivial solution. Why?
ecause there are only 3 equations (corresponding to the number of rows),
ut there are 4 variables (corresponding to the number of vectors which then become columns).
ee Section 2.2, Theorem 2.6 for more detail.

urthermore, this follows immediately from Theorem 2.8. Why?
'his is a set of 4 vectors in $\mathbb{R}^3$. Why does Theorem 2.8 apply? Hint: $4 > 3$.

'o find a specific dependence relationship, we follow Example 2.23.

'ind scalars c_1, c_2, and c_3 such that: $c_1 \begin{bmatrix} -2 \\ 3 \\ 7 \end{bmatrix} + c_2 \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix} + c_3 \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix} + c_4 \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

'orm the linear system, its associated augmented matrix, and row reduce to solve:

$$\begin{array}{rcrcrcrcl} -2c_1 & + & 4c_2 & + & 3c_3 & + & 5c_4 & = & 0 \\ 3c_1 & - & c_2 & + & c_3 & & & = & 0 \\ 7c_1 & + & 5c_2 & + & 3c_3 & + & 2c_4 & = & 0 \end{array} \Rightarrow \left[\begin{array}{rrrr|r} -2 & 4 & 3 & 5 & 0 \\ 3 & -1 & 1 & 0 & 0 \\ 7 & 5 & 3 & 2 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{rrrr|r} 37 & 0 & 0 & -13 & 0 \\ 0 & 37 & 0 & 6 & 0 \\ 0 & 0 & 37 & 45 & 0 \end{array}\right]$$

ince $c_1 = \frac{13}{37}c_4$, $c_2 = -\frac{6}{37}c_4$, $c_3 = -\frac{45}{37}c_4$ is a solution, the vectors are linearly dependent.

)ne dependence relationship is: $13 \begin{bmatrix} -2 \\ 3 \\ 7 \end{bmatrix} - 6 \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix} - 45 \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix} + 37 \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

The vectors $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ are linearly dependent.

This can be determined by inspection because $\mathbf{v}_3$ is the zero vector. Why is that enough?
Because $0\mathbf{v}_1 + 0\mathbf{v}_2 + \mathbf{v}_3 = \mathbf{0}$.
Any set of vectors containing the zero vector is linearly dependent. Why?

Since there is no obvious dependence relation here, we follow Example 2.23.

Find scalars c_1, c_2, and c_3 such that: $c_1 \begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 2 \\ 2 \\ 4 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ 3 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

Form the linear system, its associated augmented matrix, and row reduce to solve:

$$\begin{array}{rcrcrcl} -c_1 & + & 3c_2 & + & 2c_3 & = & 0 \\ c_1 & + & 2c_2 & + & 3c_3 & = & 0 \\ 2c_1 & + & c_2 & + & c_3 & = & 0 \\ c_1 & + & 4c_2 & - & c_3 & = & 0 \end{array} \Rightarrow \left[\begin{array}{rrr|r} -1 & 3 & 2 & 0 \\ 1 & 2 & 3 & 0 \\ 2 & 2 & 1 & 0 \\ 1 & 4 & -1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|r} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Since $c_1 = c_2 = c_3 = 0$ is the unique solution, the vectors are linearly independent.

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42. We will use the theorems of Sections 2.2 and 2.3 to support the conclusions below.

(a) Given $A = [\mathbf{v}_1\ \mathbf{v}_2 \ldots \mathbf{v}_n]$, we will show that $\text{rank}(A) = n$.
Theorem 2.6 of Section 2.3 implies:
Vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are linearly independent if and only if the only solution of $[A \mid 0] = [\mathbf{v}_1\ \mathbf{v}_2 \ldots \mathbf{v}_n \mid 0]$ is the trivial solution.
Therefore, the number of free variables in the associated system is 0.
So, Theorem 2.2 of Section 2.2 (The Rank Theorem) implies:
number of free variables $= 0 = n - \text{rank}(A) \Rightarrow \text{rank}(A) = n$, as claimed.

(b) Given $A = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \vdots \\ \mathbf{v}_n \end{bmatrix}$ we will show that $\text{rank}(A) = n$.
Theorem 2.7 of Section 2.3 implies:
Vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are linearly independent if and only if
the rank of $A = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \vdots \\ \mathbf{v}_n \end{bmatrix}$ is greater than or equal to n. That is, $\text{rank}(A) \geq n$.
But A has n rows therefore we have $n \leq \text{rank}(A) \leq n \Rightarrow \text{rank}(A) = n$, as claimed.

43. We apply the definition of linear independence and Examples 2.23 and 2.25 to prove our claims.

(a) We will show that $\mathbf{u}+\mathbf{v}$, $\mathbf{v}+\mathbf{w}$, and $\mathbf{u}+\mathbf{w}$ are linearly independent.
Given $c_1(\mathbf{u}+\mathbf{v}) + c_2(\mathbf{v}+\mathbf{w}) + c_3(\mathbf{u}+\mathbf{w}) = 0$, we will show $c_1 = c_2 = c_3 = 0$.
Multiplying and gathering like terms yields: $(c_1+c_3)\mathbf{u} + (c_1+c_2)\mathbf{v} + (c_2+c_3)\mathbf{w} = 0$.
Since $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are linearly independent, $c_1 + c_3 = c_1 + c_2 = c_2 + c_3 = 0$.
We create the matrix of coefficients A and row reduce to determine its rank:

$$\begin{aligned} c_1 + \ c_3 &= 0 \\ c_1 + c_2 &= 0 \\ c_2 + c_3 &= 0 \end{aligned} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

Since $\text{rank}(A) = 3$ the only solution is the trivial one, so $c_1 = c_2 = c_3 = 0$.

(b) We will show that $\mathbf{u}-\mathbf{v}$, $\mathbf{v}-\mathbf{w}$, and $\mathbf{u}-\mathbf{w}$ are linearly dependent.
Given $c_1(\mathbf{u}-\mathbf{v}) + c_2(\mathbf{v}-\mathbf{w}) + c_3(\mathbf{u}-\mathbf{w}) = 0$, we will show $c_1 = c_2 = -c_3$.
Multiplying and gathering like terms yields: $(c_1+c_3)\mathbf{u} + (-c_1+c_2)\mathbf{v} + (-c_2-c_3)\mathbf{w} = 0$.
Since $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are linearly independent, $c_1 + c_3 = -c_1 + c_2 = -c_2 - c_3 = 0$.
We form the augmented matrix and row reduce to solve:

$$\begin{aligned} c_1 + \ c_3 &= 0 \\ -c_1 + c_2 &= 0 \\ -c_2 - c_3 &= 0 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

This clearly has the solution $c_1 = c_2 = -c_3$ as we were to show.

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