13. Following Example 1.24, we realize we need to find two direction vectors, \( u \) and \( v \).

Since \( P = (1, 1, 1) \), \( Q = (4, 0, 2) \), and \( R = (0, 1, -1) \) lie in plane \( \mathcal{P} \), we compute:

\[
\mathbf{u} = \overrightarrow{PQ} = q - p = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \overrightarrow{PR} = r - p = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}.
\]

Since \( u \) and \( v \) are not scalar multiples of each other, they will serve as direction vectors.

If \( u \) and \( v \) were scalar multiples of each other, we would not have a plane but simply a line.

Therefore, we have the vector equation of \( \mathcal{P} \):

\[
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}.
\]

14. Following Example 1.24, we realize we need to find two direction vectors, \( u \) and \( v \).

Since \( P = (1, 0, 0) \), \( Q = (0, 1, 0) \), and \( R = (0, 0, 1) \) lie in plane \( \mathcal{P} \), we compute:

\[
\mathbf{u} = \overrightarrow{PQ} = q - p = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \overrightarrow{PR} = r - p = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.
\]

Since \( u \) and \( v \) are not scalar multiples of each other, they will serve as direction vectors.

If \( u \) and \( v \) were scalar multiples of each other, we would not have a plane but simply a line.

Therefore, we have the vector equation of \( \mathcal{P} \):

\[
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.
\]

15. The parametric equations and associated vector forms \( x = p + td \) found below are not unique.

(a) As in the remarks prior to Example 1.20, we begin by letting \( x = t \).

When we substitute \( x = t \) into \( y = 3x - 1 \), we get \( y = 3(t) - 1 \). So, we have the following:

Parametric equations \( x = t \) and vector form \( \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t \\ -1 + 3t \end{bmatrix} \).

(b) In this case since the coefficient of \( y \) is 2, we begin by letting \( x = 2t \).

When we substitute \( x = 2t \) into \( 3x + 2y = 5 \), we get \( 3(2t) + 2y = 5 \).

Solving for \( y \) yields \( y = -3t + 2.5 \). So, we have the following:

Parametric equations: \( x = 2t \) and vector form \( \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2.5 - 3t \end{bmatrix} + t \begin{bmatrix} 2 \\ -3 \end{bmatrix} \).

We discover the following pattern: if line \( \ell \) has equation \( ax + by = c \), then \( d = \begin{bmatrix} -b \\ a \end{bmatrix} \).
5. Following Example 1.21, we will:
(a) find the vector form by substituting into \( \mathbf{x} = \mathbf{p} + t\mathbf{d} \) and
(b) find the parametric form by equating components.
\[
\begin{align*}
\text{(a)} \quad \mathbf{x} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \mathbf{p} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \text{and} \quad \mathbf{d} = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix} \Rightarrow \text{The vector form is} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}.
\end{align*}
\]
\[
\begin{align*}
\text{(b) The vector form in (a) implies the parametric form is} \quad & x = t \\
& z = 4t.
\end{align*}
\]

6. Following Example 1.21, we will:
(a) find the vector form by substituting into \( \mathbf{x} = \mathbf{p} + t\mathbf{d} \) and
(b) find the parametric form by equating components.
\[
\begin{align*}
\text{(a)} \quad \mathbf{x} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \mathbf{p} = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, \quad \text{and} \quad \mathbf{d} = \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix} \Rightarrow \text{The vector form is} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix} + t \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}.
\end{align*}
\]
\[
\begin{align*}
\text{(b) The vector form in (a) implies the parametric form is} \quad & x = 3 \\
& z = -2 + 5t.
\end{align*}
\]

7. Following Example 1.23, we will:
(a) find the normal form by substituting into \( \mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p} \) and
(b) find the general form by computing those dot products.
\[
\begin{align*}
\text{(a)} \quad \mathbf{n} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \mathbf{p} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \text{The normal form is} \quad \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 2.
\end{align*}
\]
\[
\begin{align*}
\text{(b)} \quad \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 3x + 2y + z \quad \text{and} \quad \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 2 \Rightarrow \text{The general form is} \quad 3x + 2y + z = 2.
\end{align*}
\]

8. Following Example 1.23, we will:
(a) find the normal form by substituting into \( \mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p} \) and
(b) find the general form by computing those dot products.
\[
\begin{align*}
\text{(a)} \quad \mathbf{n} = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \mathbf{p} = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} \Rightarrow \text{Normal form} \quad \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} = -3.
\end{align*}
\]
\[
\begin{align*}
\text{(b)} \quad \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x - y + 5z, \quad \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} = -3 \Rightarrow \text{The general form is} \quad x - y + 5z = -3.
\end{align*}
\]
17. Need to show \( \ell_1 \) with slope \( m_1 \) is perpendicular to \( \ell_2 \) with slope \( m_2 \) if and only if \( m_1 m_2 = -1 \). By definition, one possible form of the general equation for \( \ell_1 \) with slope \( m_1 \) is \(-m_1 x + y = b_1\). So, the normal vector for \( \ell_1 \) is \( n_1 = \begin{bmatrix} -m_1 \\ 1 \end{bmatrix} \) and the normal vector for \( \ell_2 \) is \( n_2 = \begin{bmatrix} -m_2 \\ 1 \end{bmatrix} \). Now we note \( \ell_1 \) is perpendicular to line \( \ell_2 \) if and only if \( n_1 \cdot n_2 = 0 \), so we have:

\[
n_1 \cdot n_2 = \begin{bmatrix} -m_1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -m_1 \\ 1 \end{bmatrix} = m_1 m_2 + 1 = 0 \text{ which implies } m_1 m_2 = -1 \text{ as we were to show.}
\]

18. Given \( d \) is the direction vector of line \( \ell \) and \( n \) is the normal vector to the plane \( \mathcal{P} \), we have:

If \( d \) and \( n \) are orthogonal which implies \( d \cdot n = 0 \), then line \( \ell \) is parallel to plane \( \mathcal{P} \).

If \( d \) and \( n \) are parallel which implies \( d = cn \) (scalar multiples), then \( \ell \) is perpendicular to \( \mathcal{P} \).

(a) Since the general form of \( \mathcal{P} \) is \( 2x + 3y - z = 1 \), its normal vector is \( n = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \). Since \( d = 1n = 1 \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} = d \).

(b) Since the general form of \( \mathcal{P} \) is \( 4x - y + 5z = 0 \), its normal vector is \( n = \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix} \).

Since \( d \cdot n = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix} = 2 \cdot 4 + 3 \cdot (-1) + (-1) \cdot 5 = 0 \), \( \ell \) is parallel to \( \mathcal{P} \).

(c) Since the general form of \( \mathcal{P} \) is \( x - y - z = 3 \), its normal vector is \( n = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \).

Since \( d \cdot n = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = 2 \cdot 1 + 3 \cdot (-1) + (-1) \cdot (-1) = 0 \), \( \ell \) is parallel to \( \mathcal{P} \).

(d) Since the general form of \( \mathcal{P} \) is \( 4x + 6y - 2z = 0 \), its normal vector is \( n = \begin{bmatrix} 4 \\ 6 \\ -2 \end{bmatrix} \).

Since \( d = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 4 \\ 6 \\ -2 \end{bmatrix} = \frac{1}{2} n \), \( \ell \) is perpendicular to \( \mathcal{P} \).
We will follow Example 1.26, then use \(d(Q, \mathcal{P}) = \frac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}\) and compare results.

By definition \(ax + by + cz = d\) implies \(n = [a, b, c]\), so \(x + y - z = 0\) implies \(n = [1, 1, -1]\).

As suggested by Figure 1.64, we need to calculate the length of \(\overrightarrow{PQ} = \text{proj}_n(v)\), where \(v = \overrightarrow{PQ}\).

Step 1. By trial and error, we find \(P = (1, 0, 1)\) satisfies \(x + y - z = 0\).

\[
\text{Step 2. } v = \overrightarrow{PQ} = q - p = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.
\]

Step 3. \(\text{proj}_n(v) = \left(\frac{n \cdot v}{d \cdot n}\right) n = \left(\frac{1 \cdot 1 + 1 \cdot 2 - 1 \cdot 1}{1^2 + 1^2 + (-1)^2}\right) \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 2/3 \\ -2/3 \end{bmatrix}.
\]

Step 4. The distance from \(Q\) to \(\mathcal{P}\) is \(\|\text{proj}_n(v)\| = \sqrt{\frac{2}{3}} = \frac{2\sqrt{3}}{3}\).

Now for \(d(Q, \mathcal{P}) = \frac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}\) we need identify \(a, b, c, d,\) and \(x_0, y_0, z_0\).

Since \(x + y - z = 0, a = 1, b = 1, c = -1, d = 0\). From \(Q = (2, 2, 2), x_0 = y_0 = z_0 = 2\).

So \(d(Q, \mathcal{P}) = \frac{|2 + 2 + 2 - 0|}{\sqrt{1^2 + 1^2 + (-1)^2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}\) as we found by following Example 1.26.

30. We will follow Example 1.26, then use \(d(Q, \mathcal{P}) = \frac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}\) and compare results.

By definition \(ax + by + cz = d\) implies \(n = [a, b, c]\), so \(x - 2y + 2z = 1\) implies \(n = [1, -2, 2]\).

As suggested by Figure 1.64, we need to calculate the length of \(\overrightarrow{PQ} = \text{proj}_n(v)\), where \(v = \overrightarrow{PQ}\).

Step 1. By trial and error, we find \(P = (1, 0, 0)\) satisfies \(x - 2y + 2z = 1\).

\[
\text{Step 2. } v = \overrightarrow{PQ} = q - p = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}.
\]

Step 3. \(\text{proj}_n(v) = \left(\frac{n \cdot v}{d \cdot n}\right) n = \left(\frac{-1 \cdot 1 + 0 \cdot 0 + 0 \cdot 0}{1^2 + (-2)^2 + 2^2}\right) \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = -\frac{1}{9} \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1/9 \\ 2/9 \\ -2/9 \end{bmatrix}.
\]

Step 4. The distance from \(Q\) to \(\mathcal{P}\) is \(\|\text{proj}_n(v)\| = \sqrt{-1/9} = \sqrt{-1/9} = \frac{1}{3}\).

Now for \(d(Q, \mathcal{P}) = \frac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}\) we need identify \(a, b, c, d,\) and \(x_0, y_0, z_0\).

Since \(x - 2y + 2z = 1, a = 1, b = -2, c = 2, d = 1\). From \(Q = (0, 0, 0), x_0 = y_0 = z_0 = 0\).

So \(d(Q, \mathcal{P}) = \frac{|0 - 0 + 0 - 1|}{\sqrt{1^2 + (-2)^2 + 2^2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}\) as we found by following Example 1.26.
26. Finding the distance between points \(A\) and \(B\) is equivalent to finding \(d(a, b)\).

Given \(x = [x, y, z]\), \(p = [1, 0, -2]\), and \(q = [5, 2, 4]\), we have the condition \(d(x, p) = d(x, q)\).

We simplify that equation to find the condition all points \(X = (x, y, z)\) must satisfy.

\[
d(x, p) = \sqrt{(x - 1)^2 + (y - 0)^2 + (z + 2)^2} = \sqrt{(x - 5)^2 + (y - 2)^2 + (z - 4)^2} = d(x, q).
\]

Squaring both sides, we have:

\[
(x - 1)^2 + (y - 0)^2 + (z + 2)^2 = (x - 5)^2 + (y - 2)^2 + (z - 4)^2 \Rightarrow
\]

\[
(x^2 - 2x + 1) + y^2 + (z^2 + 4z + 4) = (x^2 - 10x + 25) + (y^2 - 4y + 4) + (z^2 - 8z + 16).
\]

Noting the squares cancel and combining the other like terms, we have: \(8x + 4y + 12z = 40\).

Dividing both sides by 4, we see all points \(X = (x, y, z)\) lie in the plane \(2x + y + 3z = 10\).

27. We will first follow Example 1.25, then use \(d(Q, \ell) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}\) and compare results.

Comparing \[
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
-1 \\
2
\end{bmatrix} + t \begin{bmatrix}
1 \\
-1
\end{bmatrix}
\]
to \(x = p + td\), we see \(\ell\) has \(P = (-1, 2)\) and \(d = \begin{bmatrix}
1 \\
-1
\end{bmatrix}\).

As suggested by Figure 1.63, we need to calculate the length of \(\overrightarrow{RQ}\),
where \(R\) is the point on \(\ell\) at the foot of the perpendicular from \(Q\).

Now if we let \(v = \overrightarrow{PQ}\), then \(\overrightarrow{PQ} = \text{proj}_d(v)\) and \(\overrightarrow{RQ} = v - \text{proj}_d(v)\).

Step 1. \(v = \overrightarrow{PQ} = q - p = \begin{bmatrix}
2 \\
-1
\end{bmatrix} - \begin{bmatrix}
2 \\
0
\end{bmatrix} = \begin{bmatrix}
0 \\
-1
\end{bmatrix}.
\]

Step 2. \(\text{proj}_d(v) = \left(\frac{d \cdot v}{d \cdot d}\right) d = \left(\frac{1 \cdot 3 + (-1) \cdot 0}{1 \cdot 1 + (-1) \cdot (-1)}\right) \begin{bmatrix}
1 \\
-1
\end{bmatrix} = \frac{3}{2} \begin{bmatrix}
1 \\
-1
\end{bmatrix} = \begin{bmatrix}
3/2 \\
-3/2
\end{bmatrix}.
\]

Step 3. The vector we want is \(v - \text{proj}_d(v) = \begin{bmatrix}
3 \\
0
\end{bmatrix} - \begin{bmatrix}
3/2 \\
-3/2
\end{bmatrix} = \begin{bmatrix}
3/2 \\
3/2
\end{bmatrix}.
\]

Step 4. The distance \(d(Q, \ell)\) from \(Q\) to \(\ell\) is \(\|v - \text{proj}_d(v)\| = \\left\| \begin{bmatrix}
3/2 \\
3/2
\end{bmatrix} \right\| = \frac{\sqrt{3}}{2} \frac{\sqrt{1 + 1}}{2} = \frac{3\sqrt{2}}{2}.
\]

So Theorem 1.3(b) implies \(\|v - \text{proj}_d(v)\| = \frac{3}{2} \left\| \begin{bmatrix}
1 \\
1
\end{bmatrix} \right\| = \frac{3}{2} \sqrt{1 + 1} = \frac{3\sqrt{2}}{2}.
\]

Now in order to calculate \(d(Q, \ell) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}\) we need to put \(\ell\) into general form.

If \(d = \begin{bmatrix}
a \\
b
\end{bmatrix}\), then \(n = \begin{bmatrix}
b \\
-a
\end{bmatrix}\) because \(\begin{bmatrix}
a \\
b
\end{bmatrix} \cdot \begin{bmatrix}
b \\
-a
\end{bmatrix} = 0\). For \(\ell\), \(d = \begin{bmatrix}
-1 \\
1
\end{bmatrix}\) so \(n = \begin{bmatrix}
1 \\
1
\end{bmatrix}\).

From \(n \cdot x = n \cdot p\) we have \(\begin{bmatrix}
1 \\
1
\end{bmatrix} \cdot \begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix}
1 \\
1
\end{bmatrix} \cdot \begin{bmatrix}
-1 \\
2
\end{bmatrix}\) so \(x + y = 1\) and \(a = b = c = 1\).

Furthermore, since \(Q = (2, 2) = (x_0, y_0)\) we have \(x_0 = y_0 = 2\).

So \(d(Q, \ell) = \frac{2 + 2 - 1}{\sqrt{1^2 + 1^2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}\) exactly as we found by following Example 1.25.
25. Following Example 1.23, we will determine the general equations in two simple steps:
   First, we will use Figure 1.31 in Section 1.2 to find a normal vector \( \mathbf{n} \) and a point vector \( \mathbf{p} \).
   Then we will substitute into \( \mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p} \) and compute the dot products to find the equations.

   (a) We start with \( \mathcal{P}_1 \) determined by the face of the cube in the \( yz \)-plane.

   It is clear that a normal vector for \( \mathcal{P}_1 \) is \( \mathbf{n} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \) or any vector parallel to the \( x \)-axis.

   Also we see that \( \mathcal{P}_1 \) passes through the origin \( P = (0, 0, 0) \), so we set \( \mathbf{p} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \).

   Substituting into \( \mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p} \) yields
   \[
   \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{or} \quad 1 \cdot x + 0 \cdot y + 0 \cdot z = 0.
   \]

   So, the general equation for \( \mathcal{P}_1 \) determined by the face in the \( yz \)-plane is \( x = 0 \).

   Likewise, the general equation for \( \mathcal{P}_2 \) determined by the face in the \( xz \)-plane is \( y = 0 \)
   and the general equation for \( \mathcal{P}_3 \) determined by the face in the \( xy \)-plane is \( z = 0 \).

   We have found equations for the planes that pass through the origin.

   We will use this information to find equations for the planes that pass through \( (1, 1, 1) \).

   We begin with \( \mathcal{P}_4 \) passing through the face parallel to the face in the \( yz \)-plane.

   Since \( \mathcal{P}_4 \) is parallel to the face in the \( yz \)-plane, its normal vector is \( \mathbf{n} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \).

   As previously noted \( \mathcal{P}_4 \) passes through the point \( P = (1, 1, 1) \), so we set \( \mathbf{p} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \).

   Substituting into \( \mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p} \) yields
   \[
   \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{or} \quad 1 \cdot x + 0 \cdot y + 1 \cdot z = 1.
   \]

   So, the general equation for \( \mathcal{P}_4 \) is \( x = 1 \).

   Likewise, the general equations for \( \mathcal{P}_5 \) and \( \mathcal{P}_6 \) are \( y = 1 \) and \( z = 1 \) respectively.