HW 4: 7, 14, 18, 23, 28

EC 25

13. Following Example 1.24, we realize we need to find two direction vectors, **u** and **v**. Since $P = (1,1,1)$, $Q = (4,0,2)$, and $R = (0,1,-1)$ lie in plane $\mathscr{P}$, we compute:

$$\mathbf{u} = \overrightarrow{PQ} = \mathbf{q} - \mathbf{p} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \text{ and } \mathbf{v} = \overrightarrow{PR} = \mathbf{r} - \mathbf{p} = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}.$$

Since **u** and **v** are not scalar multiples of each other, they will serve as direction vectors. If **u** and **v** were scalar multiples of each other, we would not have a plane but simply a line.

Therefore, we have the vector equation of $\mathscr{P}$: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}$.

RB

14. Following Example 1.24, we realize we need to find two direction vectors, **u** and **v**. Since $P = (1,0,0)$, $Q = (0,1,0)$, and $R = (0,0,1)$ lie in plane $\mathscr{P}$, we compute:

$$\mathbf{u} = \overrightarrow{PQ} = \mathbf{q} - \mathbf{p} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \text{ and } \mathbf{v} = \overrightarrow{PR} = \mathbf{r} - \mathbf{p} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

Since **u** and **v** are not scalar multiples of each other, they will serve as direction vectors. If **u** and **v** were scalar multiples of each other, we would not have a plane but simply a line.

Therefore, we have the vector equation of $\mathscr{P}$: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

15. The parametric equations and associated vector forms $\mathbf{x} = \mathbf{p} + t\mathbf{d}$ found below are *not* unique.

(a) As in the remarks prior to Example 1.20, we begin by letting $x = t$.
When we substitute $x = t$ into $y = 3x - 1$, we get $y = 3(t) - 1$. So, we have the following:
Parametric equations $\begin{aligned} x &= t \\ y &= -1 + 3t \end{aligned}$ and vector form $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

(b) In this case since the coefficient of y is 2, we begin by letting $x = 2t$.
When we substitute $x = 2t$ into $3x + 2y = 5$, we get $3(2t) + 2y = 5$.
Solving for y yields $y = -3t + 2.5$. So, we have the following:
Parametric equations: $\begin{aligned} x &= 2t \\ y &= 2.5 - 3t \end{aligned}$ and vector form $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2.5 \end{bmatrix} + t \begin{bmatrix} 2 \\ -3 \end{bmatrix}$.

We discover the following pattern: if line ℓ has equation $ax + by = c$, then $\mathbf{d} = \begin{bmatrix} b \\ -a \end{bmatrix}$.

16. [illegible]

5. Following Example 1.21, we will:
(a) find the vector form by substituting into $\mathbf{x} = \mathbf{p} + t\mathbf{d}$ and
(b) find the parametric form by equating components.

(a) $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, and $\mathbf{d} = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$ $\Rightarrow$ The vector form is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$.

(b) The vector form in (a) implies the parametric form is $\begin{aligned} x &= t \\ y &= -t \\ z &= 4t \end{aligned}$.

6. Following Example 1.21, we will:
(a) find the vector form by substituting into $\mathbf{x} = \mathbf{p} + t\mathbf{d}$ and
(b) find the parametric form by equating components.

(a) $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$, and $\mathbf{d} = \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$ $\Rightarrow$ The vector form is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix} + t \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$.

(b) The vector form in (a) implies the parametric form is $\begin{aligned} x &= 3 \\ y &= 2t \\ z &= -2 + 5t \end{aligned}$.

RB

7. Following Example 1.23, we will:
(a) find the normal form by substituting into $\mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p}$ and
(b) find the general form by computing those dot products.

(a) $\mathbf{n} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\Rightarrow$ The normal form is $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 2$.

(b) $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 3x + 2y + z$ and $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 2 \Rightarrow$ The general form is $3x + 2y + z = 2$.

8. Following Example 1.23, we will:
(a) find the normal form by substituting into $\mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p}$ and
(b) find the general form by computing those dot products.

(a) $\mathbf{n} = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}$ $\Rightarrow$ Normal form $\begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} =$
-3.

(b) $\begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x - y + 5z$, $\begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} = -3 \Rightarrow$ The general form is $x - y + 5z = -3$

17. Need to show ℓ_1 with slope m_1 is perpendicular to ℓ_2 with slope m_2 if and only if $m_1 m_2 = -1$.
By definition, one possible form of the general equation for ℓ_1 with slope m_1 is $-m_1 x + y = b_1$.
So, the normal vector for ℓ_1 is $\mathbf{n}_1 = \begin{bmatrix} -m_1 \\ 1 \end{bmatrix}$ and the normal vector for ℓ_2 is $\mathbf{n}_2 = \begin{bmatrix} -m_2 \\ 1 \end{bmatrix}$.
Now we note ℓ_1 is perpendicular to line ℓ_2 if and only if $\mathbf{n}_1 \cdot \mathbf{n}_2 = 0$, so we have:
$\mathbf{n}_1 \cdot \mathbf{n}_2 = \begin{bmatrix} -m_1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -m_1 \\ 1 \end{bmatrix} = m_1 m_2 + 1 = 0$ which implies $m_1 m_2 = -1$ as we were to show.

PB

18. Given $\mathbf{d}$ is the direction vector of line ℓ and $\mathbf{n}$ is the normal vector to the plane $\mathscr{P}$, we have:
If $\mathbf{d}$ and $\mathbf{n}$ are orthogonal which implies $\mathbf{d} \cdot \mathbf{n} = 0$, then line ℓ is parallel to plane $\mathscr{P}$.
If $\mathbf{d}$ and $\mathbf{n}$ are parallel which implies $\mathbf{d} = c\mathbf{n}$ (scalar multiples), then ℓ is perpendicular to $\mathscr{P}$.

(a) Since the general form of $\mathscr{P}$ is $2x + 3y - z = 1$, its normal vector is $\mathbf{n} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} = \mathbf{d}$.

Since $\mathbf{d} = 1\,\mathbf{n}$, ℓ is perpendicular to $\mathscr{P}$.

(b) Since the general form of $\mathscr{P}$ is $4x - y + 5z = 0$, its normal vector is $\mathbf{n} = \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix}$.

Since $\mathbf{d} \cdot \mathbf{n} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix} = 2 \cdot 4 + 3 \cdot (-1) + (-1) \cdot 5 = 0$, ℓ is parallel to $\mathscr{P}$.

(c) Since the general form of $\mathscr{P}$ is $x - y - z = 3$, its normal vector is $\mathbf{n} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$.

Since $\mathbf{d} \cdot \mathbf{n} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = 2 \cdot 1 + 3 \cdot (-1) + (-1) \cdot (-1) = 0$, ℓ is parallel to $\mathscr{P}$.

(d) Since the general form of $\mathscr{P}$ is $4x + 6y - 2z = 0$, its normal vector is $\mathbf{n} = \begin{bmatrix} 4 \\ 6 \\ -2 \end{bmatrix}$.

Since $\mathbf{d} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 4 \\ 6 \\ -2 \end{bmatrix} = \frac{1}{2}\mathbf{n}$, ℓ is perpendicular to $\mathscr{P}$.

RR

29. We will follow Example 1.26, then use $d(Q, \mathscr{P}) = \dfrac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2+b^2+c^2}}$ and compare results.
By definition $ax + by + cz = d$ implies $\mathbf{n} = [a, b, c]$, so $x + y - z = 0$ implies $\mathbf{n} = [1, 1, -1]$.
As suggested by Figure 1.64, we need to calculate the length of $\overrightarrow{RQ} = \text{proj}_{\mathbf{n}}(\mathbf{v})$, where $\mathbf{v} = \overrightarrow{PQ}$.
Step 1. By trial and error, we find $P = (1, 0, 1)$ satisfies $x + y - z = 0$.

Step 2. $\mathbf{v} = \overrightarrow{PQ} = \mathbf{q} - \mathbf{p} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

Step 3. $\text{proj}_{\mathbf{n}}(\mathbf{v}) = \left(\dfrac{\mathbf{n}\cdot\mathbf{v}}{\mathbf{d}\cdot\mathbf{n}}\right)\mathbf{n} = \left(\dfrac{1\cdot 1 + 1\cdot 2 - 1\cdot 1}{1^2+1^2+(-1)^2}\right)\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \dfrac{2}{3}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 2/3 \\ -2/3 \end{bmatrix}$.

Step 4. The distance from Q to $\mathscr{P}$ is $\|\text{proj}_{\mathbf{n}}(\mathbf{v})\| = \left\|\begin{bmatrix} 2/3 \\ 2/3 \\ -2/3 \end{bmatrix}\right\| = \dfrac{2}{3}\left\|\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}\right\| = \dfrac{2\sqrt{3}}{3}$.

Now for $d(Q, \mathscr{P}) = \dfrac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2+b^2+c^2}}$ we need identify a, b, c, d, and x_0, y_0, z_0.
Since $x + y - z = 0$, $a = 1$, $b = 1$, $c = -1$, $d = 0$. From $Q = (2, 2, 2)$, $x_0 = y_0 = z_0 = 2$.
So $d(Q, \mathscr{P}) = \dfrac{|2 + 2 - 2 + 0|}{\sqrt{1^2+1^2+(-1)^2}} = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$ as we found by following Example 1.26.

30. We will follow Example 1.26, then use $d(Q, \mathscr{P}) = \dfrac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2+b^2+c^2}}$ and compare results.
By definition $ax + by + cz = d$ implies $\mathbf{n} = [a, b, c]$, so $x - 2y + 2z = 1$ implies $\mathbf{n} = [1, -2, 2]$.
As suggested by Figure 1.64, we need to calculate the length of $\overrightarrow{RQ} = \text{proj}_{\mathbf{n}}(\mathbf{v})$, where $\mathbf{v} = \overrightarrow{PQ}$.
Step 1. By trial and error, we find $P = (1, 0, 0)$ satisfies $x - 2y + 2z = 1$.

Step 2. $\mathbf{v} = \overrightarrow{PQ} = \mathbf{q} - \mathbf{p} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$.

Step 3. $\text{proj}_{\mathbf{n}}(\mathbf{v}) = \left(\dfrac{\mathbf{n}\cdot\mathbf{v}}{\mathbf{d}\cdot\mathbf{n}}\right)\mathbf{n} = \left(\dfrac{-1\cdot 1 + 0\cdot 0 + 0\cdot 0}{1^2+(-2)^2+2^2}\right)\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = -\dfrac{1}{9}\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1/9 \\ 2/9 \\ -2/9 \end{bmatrix}$.

Step 4. The distance from Q to $\mathscr{P}$ is $\|\text{proj}_{\mathbf{n}}(\mathbf{v})\| = \left\|\begin{bmatrix} -1/9 \\ 2/9 \\ -2/9 \end{bmatrix}\right\| = \dfrac{1}{9}\left\|\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}\right\| = \dfrac{1}{3}$.

Now for $d(Q, \mathscr{P}) = \dfrac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2+b^2+c^2}}$ we need identify a, b, c, d, and x_0, y_0, z_0.
Since $x - 2y + 2z = 1$, $a = 1$, $b = -2$, $c = 2$, $d = 1$. From $Q = (0, 0, 0)$, $x_0 = y_0 = z_0 = 0$.
So $d(Q, \mathscr{P}) = \dfrac{|0 - 0 + 0 - 1|}{\sqrt{1^2+(-2)^2+2^2}} = \dfrac{1}{\sqrt{9}} = \dfrac{1}{3}$ as we found by following Example 1.26.

26. Finding the distance between points A and B is equivalent to finding $d(\mathbf{a}, \mathbf{b})$. Given $\mathbf{x} = [x, y, z]$, $\mathbf{p} = [1, 0, -2]$, and $\mathbf{q} = [5, 2, 4]$, we have the condition $d(\mathbf{x}, \mathbf{p}) = d(\mathbf{x}, \mathbf{q})$. We simplify that equation to find the condition all points $X = (x, y, z)$ must satisfy.

$$d(\mathbf{x}, \mathbf{p}) = \sqrt{(x-1)^2 + (y-0)^2 + (z+2)^2} = \sqrt{(x-5)^2 + (y-2)^2 + (z-4)^2} = d(\mathbf{x}, \mathbf{q}).$$

Squaring both sides, we have: $(x-1)^2 + (y-0)^2 + (z+2)^2 = (x-5)^2 + (y-2)^2 + (z-4)^2 \Rightarrow$

$$(x^2 - 2x + 1) + y^2 + (z^2 + 4z + 4) = (x^2 - 10x + 25) + (y^2 - 4y + 4) + (z^2 - 8z + 16).$$

Noting the squares cancel and combining the other like terms, we have: $8x + 4y + 12z = 40$. Dividing both sides by 4, we see all points $X = (x, y, z)$ lie in the plane $2x + y + 3z = 10$.

R **27.** We will first follow Example 1.25, then use $d(Q, \ell) = \dfrac{|ax_0 + by_0 - c|}{\sqrt{a^2 + b^2}}$ and compare results.

Comparing $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ to $\mathbf{x} = \mathbf{p} + t\mathbf{d}$, we see ℓ has $P = (-1, 2)$ and $\mathbf{d} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

As suggested by Figure 1.63, we need to calculate the length of $\overrightarrow{RQ}$, where R is the point on ℓ at the foot of the perpendicular from Q.

Now if we let $\mathbf{v} = \overrightarrow{PQ}$, then $\overrightarrow{PR} = \text{proj}_{\mathbf{d}}(\mathbf{v})$ and $\overrightarrow{RQ} = \mathbf{v} - \text{proj}_{\mathbf{d}}(\mathbf{v})$.

Step 1. $\mathbf{v} = \overrightarrow{PQ} = \mathbf{q} - \mathbf{p} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$.

Step 2. $\text{proj}_{\mathbf{d}}(\mathbf{v}) = \left(\dfrac{\mathbf{d} \cdot \mathbf{v}}{\mathbf{d} \cdot \mathbf{d}}\right) \mathbf{d} = \left(\dfrac{1 \cdot 3 + (-1) \cdot 0}{1 \cdot 1 + (-1) \cdot (-1)}\right) \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \dfrac{3}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3/2 \\ -3/2 \end{bmatrix}$.

Step 3. The vector we want is $\mathbf{v} - \text{proj}_{\mathbf{d}}(\mathbf{v}) = \begin{bmatrix} 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 3/2 \\ -3/2 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 3/2 \end{bmatrix}$.

Step 4. The distance $d(Q, \ell)$ from Q to ℓ is $\|\mathbf{v} - \text{proj}_{\mathbf{d}}(\mathbf{v})\| = \left\| \begin{bmatrix} 3/2 \\ 3/2 \end{bmatrix} \right\|$.

So Theorem 1.3(b) implies $\|\mathbf{v} - \text{proj}_{\mathbf{d}}(\mathbf{v})\| = \dfrac{3}{2} \left\| \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\| = \dfrac{3}{2}\sqrt{1+1} = \dfrac{3\sqrt{2}}{2}$.

Now in order to calculate $d(Q, \ell) = \dfrac{|ax_0 + by_0 - c|}{\sqrt{a^2 + b^2}}$ we need to put ℓ into general form.

If $\mathbf{d} = \begin{bmatrix} a \\ b \end{bmatrix}$, then $\mathbf{n} = \begin{bmatrix} b \\ -a \end{bmatrix}$ because $\begin{bmatrix} a \\ b \end{bmatrix} \cdot \begin{bmatrix} b \\ -a \end{bmatrix} = 0$. For ℓ, $\mathbf{d} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ so $\mathbf{n} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

From $\mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p}$ we have $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ so $x + y = 1$ and $a = b = c = 1$.

Furthermore, since $Q = (2, 2) = (x_0, y_0)$ we have $x_0 = y_0 = 2$.

So $d(Q, \ell) = \dfrac{|2 + 2 - 1|}{\sqrt{1^2 + 1^2}} = \dfrac{3}{\sqrt{2}} = \dfrac{3\sqrt{2}}{2}$ exactly as we found by following Example 1.25.

25. Following Example 1.23, we will determine the general equations in two simple steps: First, we will use Figure 1.31 in Section 1.2 to find a normal vector $\mathbf{n}$ and a point vector $\mathbf{p}$. Then we will substitute into $\mathbf{n}\cdot\mathbf{x} = \mathbf{n}\cdot\mathbf{p}$ and compute the dot products to find the equations.

(a) We start with $\mathscr{P}_1$ determined by the face of the cube in the yz-plane.

It is clear that a normal vector for $\mathscr{P}_1$ is $\mathbf{n} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ or any vector parallel to the x-axis.

Also we see that $\mathscr{P}_1$ passes through the origin $P = (0,0,0)$, so we set $\mathbf{p} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

Substituting into $\mathbf{n}\cdot\mathbf{x} = \mathbf{n}\cdot\mathbf{p}$ yields $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ or $1\cdot x + 0\cdot y + 0\cdot z = 0$.

So, the general equation for $\mathscr{P}_1$ determined by the face in the yz-plane is $x = 0$.
Likewise, the general equation for $\mathscr{P}_2$ determined by the face in the xz-plane is $y = 0$ and the general equation for $\mathscr{P}_3$ determined by the face in the xy-plane is $z = 0$.
We have found equations for the planes that pass through the origin.
We will use this information to find equations for the planes that pass through $(1,1,1)$.
We begin with $\mathscr{P}_4$ passing through the face parallel to the face in the yz-plane.

Since $\mathscr{P}_4$ is parallel to the face in the yz-plane, its normal vector is $\mathbf{n} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

As previously noted $\mathscr{P}_4$ passes through the point $P = (1,1,1)$, so we set $\mathbf{p} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Substituting into $\mathbf{n}\cdot\mathbf{x} = \mathbf{n}\cdot\mathbf{p}$ yields $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ or $1\cdot x + 0\cdot y + 0\cdot z = 1$.

So, the general equation for $\mathscr{P}_4$ is $x = 1$.
Likewise, the general equations for $\mathscr{P}_5$ and $\mathscr{P}_6$ are $y = 1$ and $z = 1$ respectively.