HW4: 1, 4, 5, 11, 12, 20

1.3 Lines and Planes

PB 1. Following Example 1.20, we will:
(a) find the normal form by substituting into $\mathbf{n}\cdot\mathbf{x}=\mathbf{n}\cdot\mathbf{p}$ and
(b) find the general form by computing those dot products.

(a) $\mathbf{n}=\begin{bmatrix}3\\2\end{bmatrix}$, $\mathbf{x}=\begin{bmatrix}x\\y\end{bmatrix}$, and $\mathbf{p}=\begin{bmatrix}0\\0\end{bmatrix}$ $\Rightarrow$ The normal form is $\begin{bmatrix}3\\2\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3\\2\end{bmatrix}\cdot\begin{bmatrix}0\\0\end{bmatrix}=0$.

(b) $\begin{bmatrix}3\\2\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=3x+2y$ and $\begin{bmatrix}3\\2\end{bmatrix}\cdot\begin{bmatrix}0\\0\end{bmatrix}=0 \Rightarrow$ The general form is $3x+2y=0$.

2. Following Example 1.20, we will:
(a) find the normal form by substituting into $\mathbf{n}\cdot\mathbf{x}=\mathbf{n}\cdot\mathbf{p}$ and
(b) find the general form by computing those dot products.

(a) $\mathbf{n}=\begin{bmatrix}5\\-3\end{bmatrix}$, $\mathbf{x}=\begin{bmatrix}x\\y\end{bmatrix}$, $\mathbf{p}=\begin{bmatrix}1\\2\end{bmatrix}$ $\Rightarrow$ Normal form is $\begin{bmatrix}5\\-3\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}5\\-3\end{bmatrix}\cdot\begin{bmatrix}1\\2\end{bmatrix}=-1$.

(b) $\begin{bmatrix}5\\-3\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=5x-3y$ and $\begin{bmatrix}5\\-3\end{bmatrix}\cdot\begin{bmatrix}1\\2\end{bmatrix}=0 \Rightarrow$ The general form is $5x-3y=-1$.

3. Following Example 1.21, we will:
(a) find the vector form by substituting into $\mathbf{x}=\mathbf{p}+t\mathbf{d}$ and
(b) find the parametric form by equating components.

(a) $\mathbf{x}=\begin{bmatrix}x\\y\end{bmatrix}$, $\mathbf{p}=\begin{bmatrix}1\\0\end{bmatrix}$, and $\mathbf{d}=\begin{bmatrix}-1\\3\end{bmatrix}$ $\Rightarrow$ The vector form is $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}+t\begin{bmatrix}-1\\3\end{bmatrix}$.

PB (b) The vector form in (a) implies the parametric form is $\begin{matrix}x=1-t\\y=3t\end{matrix}$.

4. Following Example 1.21, we will:
(a) find the vector form by substituting into $\mathbf{x}=\mathbf{p}+t\mathbf{d}$ and
(b) find the parametric form by equating components.

(a) $\mathbf{x}=\begin{bmatrix}x\\y\end{bmatrix}$, $\mathbf{p}=\begin{bmatrix}-4\\4\end{bmatrix}$, and $\mathbf{d}=\begin{bmatrix}1\\1\end{bmatrix}$ $\Rightarrow$ The vector form is $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-4\\4\end{bmatrix}+t\begin{bmatrix}1\\1\end{bmatrix}$.

(b) The vector form in (a) implies the parametric form is $\begin{matrix}x=-4+t\\y=4+t\end{matrix}$.

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5. Following Example 1.21, we will:
(a) find the vector form by substituting into $\mathbf{x} = \mathbf{p} + t\mathbf{d}$ and
(b) find the parametric form by equating components.

(a) $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, and $\mathbf{d} = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$ $\Rightarrow$ The vector form is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$.

(b) The vector form in (a) implies the parametric form is $\begin{aligned} x &= t \\ y &= -t \\ z &= 4t \end{aligned}$.

6. Following Example 1.21, we will:
(a) find the vector form by substituting into $\mathbf{x} = \mathbf{p} + t\mathbf{d}$ and
(b) find the parametric form by equating components.

(a) $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$, and $\mathbf{d} = \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$ $\Rightarrow$ The vector form is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix} + t \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$.

(b) The vector form in (a) implies the parametric form is $\begin{aligned} x &= 3 \\ y &= 2t \\ z &= -2 + 5t \end{aligned}$.

7. Following Example 1.23, we will:
(a) find the normal form by substituting into $\mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p}$ and
(b) find the general form by computing those dot products.

(a) $\mathbf{n} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\Rightarrow$ The normal form is $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 2$.

(b) $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 3x + 2y + z$ and $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 2 \Rightarrow$ The general form is $3x + 2y + z = 2$.

8. Following Example 1.23, we will:
(a) find the normal form by substituting into $\mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{p}$ and
(b) find the general form by computing those dot products.

(a) $\mathbf{n} = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}$ $\Rightarrow$ Normal form $\begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} =$ -3.

(b) $\begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x - y + 5z$, $\begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix} = -3 \Rightarrow$ The general form is $x - y + 5z = -3$.

9. Following Example 1.24, we will:
(a) find the vector form by substituting into $\mathbf{x} = \mathbf{p} + s\mathbf{u} + t\mathbf{v}$ and
(b) find the parametric form by equating components.

(a) $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, and $\mathbf{v} = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} \Rightarrow$

The vector form is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + t \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$.

(b) The vector form in (a) implies the parametric form is $\begin{array}{l} x = 2s - 3t \\ y = s + 2t \\ z = 2s + t \end{array}$.

10. Following Example 1.24, we will:
(a) find the vector form by substituting into $\mathbf{x} = \mathbf{p} + s\mathbf{u} + t\mathbf{v}$ and
(b) find the parametric form by equating components.

(a) $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} 6 \\ -4 \\ -3 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, and $\mathbf{v} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow$

The vector form is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ -3 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$.

(b) The vector form in (a) implies the parametric form is $\begin{array}{l} x = 6 - \phantom{s+{}} t \\ y = -4 + s + t \\ z = -3 + s + t \end{array}$.

PB

11. Following Example 1.24, we realize we may choose any point on ℓ, so we will use P (Q would also be fine).

A convenient direction vector is $\mathbf{d} = \overrightarrow{PQ} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ (or any scalar multiple of this).

Thus we obtain:
$$\begin{aligned} \mathbf{x} &= \mathbf{p} + t\mathbf{d} \\ &= \begin{bmatrix} 1 \\ -2 \end{bmatrix} + t \begin{bmatrix} 2 \\ 2 \end{bmatrix}. \end{aligned}$$

12. Following Example 1.24, we realize we may choose any point on ℓ, so we will use P (Q would also be fine).

A convenient direction vector is $\mathbf{d} = \overrightarrow{PQ} = \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}$ (or any scalar multiple of this).

Thus we obtain:
$$\begin{aligned} \mathbf{x} &= \mathbf{p} + t\mathbf{d} \\ &= \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + t \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}. \end{aligned}$$

19. Given $\mathbf{n}_1$ is the normal vector of $\mathscr{P}_1$ and $\mathbf{n}$ is the normal vector of $\mathscr{P}$, we have:
If $\mathbf{n}_1$ and $\mathbf{n}$ are orthogonal which implies $\mathbf{n}_1 \cdot \mathbf{n} = 0$, then $\mathscr{P}_1$ is perpendicular to $\mathscr{P}$.
If $\mathbf{n}_1$ and $\mathbf{n}$ are parallel which implies $\mathbf{n}_1 = c\mathbf{n}$ (scalar multiples), then $\mathscr{P}_1$ is parallel to $\mathscr{P}$.

(a) Since the general form of $\mathscr{P}$ is $2x + 3y - z = 1$, its normal vector is $\mathbf{n} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$.

Since $\mathbf{n}_1 \cdot \mathbf{n} = \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} = 4 \cdot 2 + (-1) \cdot 3 + 5 \cdot (-1) = 0$, $\mathscr{P}_1$ is perpendicular to $\mathscr{P}$.

(b) Since the general form of $\mathscr{P}$ is $4x - y + 5z = 0$, its normal vector is $\mathbf{n} = \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix}$.

Since $\mathbf{n}_1 = 1\,\mathbf{n}$, $\mathscr{P}_1$ is parallel to $\mathscr{P}$.

(c) Since the general form of $\mathscr{P}$ is $x - y - z = 3$, its normal vector is $\mathbf{n} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$.

Since $\mathbf{n}_1 \cdot \mathbf{n} = \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = 0$, $\mathscr{P}_1$ is perpendicular to $\mathscr{P}$.

(d) Since the general form of $\mathscr{P}$ is $4x + 6y - 2z = 0$, its normal vector is $\mathbf{n} = \begin{bmatrix} 4 \\ 6 \\ -2 \end{bmatrix}$.

Since $\mathbf{n}_1 \cdot \mathbf{n} = \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 6 \\ -2 \end{bmatrix} = 0$, $\mathscr{P}_1$ is perpendicular to $\mathscr{P}$.

RB

20. Since the vector form is $\mathbf{x} = \mathbf{p} + t\mathbf{d}$, we use the given information to determine $\mathbf{p}$ and $\mathbf{d}$.

The general equation of the given line is $2x - 3y = 1$, so its normal vector is $\mathbf{n} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$.

Our line is perpendicular to the given line, so it has direction vector $\mathbf{d} = \mathbf{n} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$.

Furthermore, since our line passes through the point $P = (2, -1)$, we have $\mathbf{p} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

So, the vector form of the line perpendicular to $2x - 3y = 1$ through the point $P = (2, -1)$ is

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} + t \begin{bmatrix} 2 \\ -3 \end{bmatrix}.$$