HW #1

1.1 The Geometry and Algebra of Vectors

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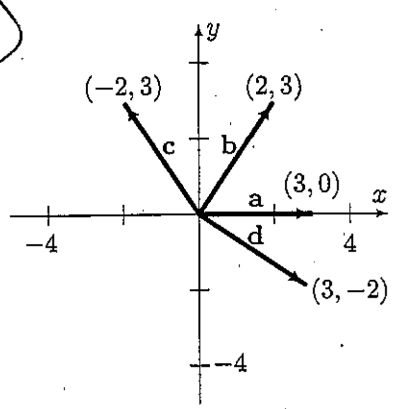
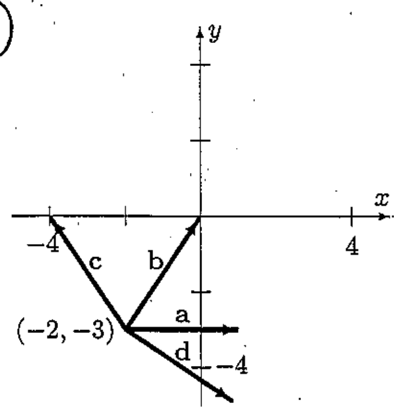


2\.



3\. See Figures 1.14 and 1.15.

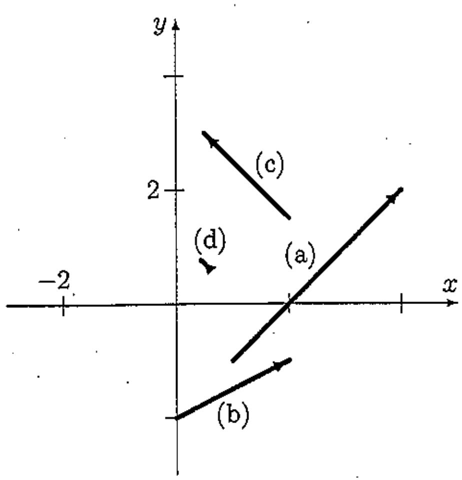
4\. (a) Following Example 1.1, we have the following:
If $[0, 2, 0]$ is translated to $\overrightarrow{BC}$ where $C = (4, 5, 6)$,
then we must have $B = (4 - 0, 5 - 2, 6 - 0) = (4, 3, 6)$.
Note: Unlike Example 1.1, we subtract $[0, 2, 0]$ instead of adding $[0, 2, 0]$. Why?

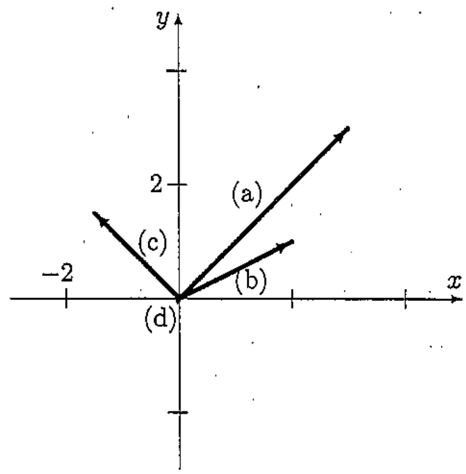
(b) Likewise, if $[3, 2, 1]$ is translated, $B = (4 - 3, 5 - 2, 6 - 1) = (1, 3, 5)$.

(c) If $[1, -2, 1]$ is translated, $B = (4 - 1, 5 - (-2), 6 - 1) = (3, 7, 5)$.

(d) If $[-1, -1, -2]$ is translated, $B = (4 - (-1), 5 - (-1), 6 - (-2)) = (5, 6, 8)$.

5\. (a) $\overrightarrow{AB} = [4 - 1, 2 - (-1)] = [3, 3]$.
(b) $[2, 1]$ (c) $[-\frac{3}{2}, \frac{3}{2}]$ (d) $[-\frac{1}{6}, \frac{1}{6}]$.



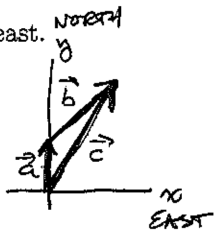


§1.1

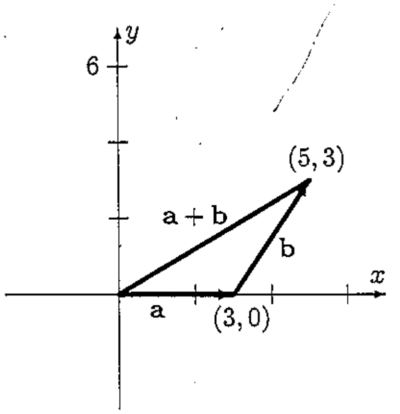
6. Recall the notation that $[a, b]$ denotes a move of a units horizontally and b units vertically.
During the first part of the walk, the hiker walks 4 km north, so $\mathbf{a} = [0, 4]$.
During the second part of the walk, the hiker walks a distance of 5 km northeast.

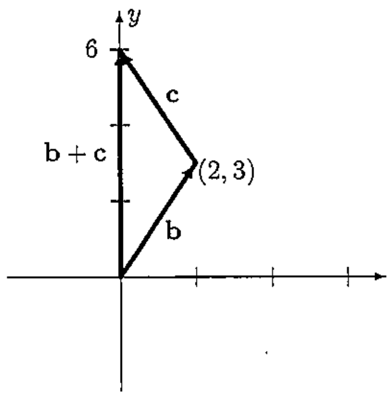
From the components, we get $\mathbf{b} = [5\cos 45°, 5\sin 45°] = \left[\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right]$.

Thus, the net displacement vector is $\mathbf{c} = \mathbf{a} + \mathbf{b} = \left[\frac{5\sqrt{2}}{2}, 4 + \frac{5\sqrt{2}}{2}\right]$.



7. $\mathbf{a} + \mathbf{b} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3+2 \\ 0+3 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$ **8.** $\mathbf{b} + \mathbf{c} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$





9. $\mathbf{d} - \mathbf{c} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} - \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$. **10.** $\mathbf{a} - \mathbf{d} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$.

11. $2\mathbf{a} + 3\mathbf{c} = 2\,[0, 2, 0] + 3\,[1, -2, 1] = [2 \cdot 0, 2 \cdot 2, 2 \cdot 0] + [3 \cdot 1, 3(-2), 3 \cdot 1] = [3, -2, 3]$.

12. $2\mathbf{c} - 3\mathbf{b} - \mathbf{d} = 2\,[1, -2, 1] - 3\,[3, 2, 1] - [-1, -1, -2] = [-6, -9, 1]$.

13. $\mathbf{u} = [\cos 60°, \sin 60°] = \left[\frac{1}{2}, \frac{\sqrt{3}}{2}\right]$, $\mathbf{v} = [\cos 210°, \sin 210°] = \left[-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right] \Rightarrow$ (implies)

$\mathbf{u} + \mathbf{v} = \left[\frac{1}{2} - \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} - \frac{1}{2}\right]$, $\mathbf{u} - \mathbf{v} = \left[\frac{1}{2} + \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} + \frac{1}{2}\right]$.

14. (a) $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.

(b) $\overrightarrow{BC} = \overrightarrow{OC} - \mathbf{b} = (\mathbf{b} - \mathbf{a}) - \mathbf{b} = -\mathbf{a}$.

(c) $\overrightarrow{AD} = -2\mathbf{a}$.

(d) $\overrightarrow{CF} = \overrightarrow{CB} + \overrightarrow{BA} + \overrightarrow{AF} = -\overrightarrow{BC} - \overrightarrow{AB} + \left(-\overrightarrow{AB} - \mathbf{a}\right) - \overrightarrow{AB} = 2\,(\mathbf{a} - \mathbf{b})$.

(e) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = (\mathbf{b} - \mathbf{a}) + (-\mathbf{a}) = \mathbf{b} - 2\mathbf{a}$.

(f) $\overrightarrow{BC} + \overrightarrow{DE} + \overrightarrow{FA} = -\mathbf{a} + \left(-\overrightarrow{AB}\right) + \left(\overrightarrow{AB} + \mathbf{a}\right) = \mathbf{0}$.

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17. x -

18. x -

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19.

21. See

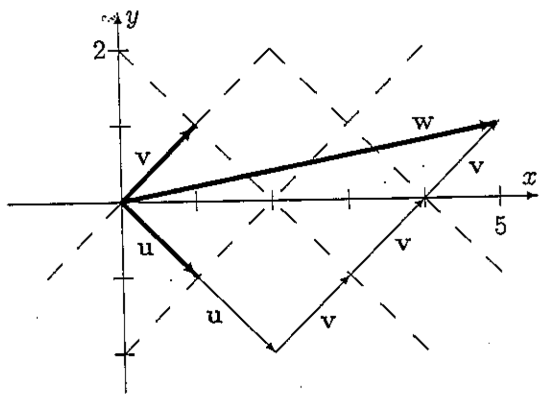
15. $2(\mathbf{a}-3\mathbf{b})+3(2\mathbf{b}+\mathbf{a}) \overset{\substack{\text{property e.}\\ \text{distributivity}}}{=} (2\mathbf{a}-6\mathbf{b})+(6\mathbf{b}+3\mathbf{a}) \overset{\substack{\text{property b.}\\ \text{associativity}}}{=} (2\mathbf{a}+3\mathbf{a})+(-6\mathbf{b}+6\mathbf{b}) = 5\mathbf{a}.$

16. $-3(\mathbf{a}-\mathbf{c})+2(\mathbf{a}+2\mathbf{b})+3(\mathbf{c}-\mathbf{b}) \overset{\substack{\text{property e.}\\ \text{distributivity}}}{=} (-3\mathbf{a}+3\mathbf{c})+(2\mathbf{a}+4\mathbf{b})+(3\mathbf{c}-3\mathbf{b}) \overset{\substack{\text{property b.}\\ \text{associativity}}}{=}$
$(-3\mathbf{a}+2\mathbf{a})+(4\mathbf{b}-3\mathbf{b})+(3\mathbf{c}+3\mathbf{c}) = -\mathbf{a}+\mathbf{b}+6\mathbf{c}.$

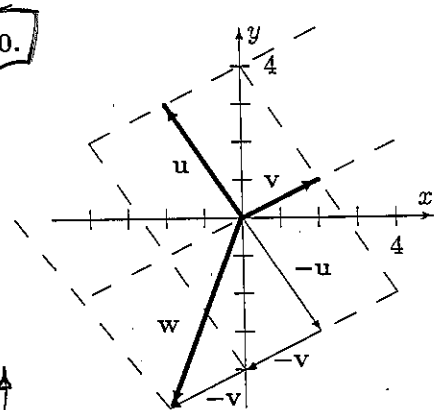
17. $\mathbf{x}-\mathbf{a} = 2(\mathbf{x}-2\mathbf{a}) = 2\mathbf{x}-4\mathbf{a} \Rightarrow \mathbf{x}-2\mathbf{x} = \mathbf{a}-4\mathbf{a} \Rightarrow -\mathbf{x} = -3\mathbf{a} \Rightarrow \mathbf{x} = 3\mathbf{a}.$

18. $\mathbf{x}+2\mathbf{a}-\mathbf{b} = 3(\mathbf{x}+\mathbf{a})-2(2\mathbf{a}-\mathbf{b}) = 3\mathbf{x}+3\mathbf{a}-4\mathbf{a}+2\mathbf{b} \Rightarrow \mathbf{x}-3\mathbf{x} = -\mathbf{a}-2\mathbf{a}+2\mathbf{b}+\mathbf{b} \Rightarrow$
$-2\mathbf{x} = -3\mathbf{a}+3\mathbf{b} \Rightarrow \mathbf{x} = \frac{3}{2}\mathbf{a}-\frac{3}{2}\mathbf{b}.$

19.



20.



21. See Exercise 19.

22. See Exercise 20.

$\vec{w} = 2\vec{u} + 3\vec{v}$