Mathematics 214: Linear Systems
Fall 2006
Test 1: Chapters 1 and 2

Name: ____________________________________

By signing below, you are agreeing on your honor that you are fully complying with the guidelines for this test and are not in any way violating the College's spirit of honor. In particular, you have not received or given any assistance on this test, nor will you give any assistance to those taking the test later.

Signature: ____________________________________

Please show your work on the problems neatly and completely. You must show the work you did and not just your answers. Please clearly identify your final answers.

No.	Score	Maximum
1		26
2		20
3		8
4		26
5		20
Total		**100**

1. [26 points] The following questions deal with these vectors in $\mathbf{R}^3$:

$$\mathbf{a} = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}, \qquad \mathbf{c} = \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}, \qquad \mathbf{d} = \begin{bmatrix} -2 \\ 2 \\ -5 \end{bmatrix}$$

(a) Find the dot product $\mathbf{a} \cdot \mathbf{b}$.

(b) Find the angle between the vectors $\mathbf{c}$ and $\mathbf{d}$.

(c) Find the length of the vector $\mathbf{b}$.

(d) Is the set of vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$ linearly independent? [**Fully explain how you arrived at your answer.**]

(e) Find the projection of the vector $\mathbf{b}$ onto $\mathbf{c}$.

(f) Give an equation of a plane in $\mathbf{R}^3$ where $\mathbf{d}$ is a point on the plane and $\mathbf{a}$ is a normal vector to the plane.

(g) How many solutions does the homogeneous system $\mathbf{a}x + \mathbf{b}y + \mathbf{c}z + \mathbf{d}w = \mathbf{0}$ have? [**Fully explain how you arrived at your answer.**]

2. [20 points] Solve the following system of linear equations by setting up the appropriate augmented matrix, finding the **reduced** row echelon form of the matrix, and then writing down the solution.

$$\begin{array}{rcrcrcr} & & 2\,y & + & 3\,z & = & 8 \\ 2\,x & + & 3\,y & + & z & = & 5 \\ x & - & y & - & 2\,z & = & -5 \end{array}$$

3. [8 points] Below are the vectors $\mathbf{u}$ and $\mathbf{v}$. Carefully draw and clearly label the vectors $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$

4. [26 points] Take as given that the reduced row echelon form of the matrix

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \qquad \text{is} \qquad \mathrm{rref}(A) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}.$$

(a) What is the rank of A? [**Explain how you know this is the answer.**]

(b) Discuss the linear independence/dependence of the columns of the matrix A. [**You must not just say they are linearly dependent or independent; you must also explain the justification for this answer.**]

(c) What is the span of the columns of matrix A? [**You must explain how you arrived at your answer. If you can give an algebraic equation for the span, you must do so.**]

5. [20 points] Consider the object in $\mathbf{R}^3$ described parametrically by $x = -3t + 1$, $y = 2t - 3$, $z = t$.

(a) What is this object? In other words, what does the object look like geometrically. **[No additional explanation needed.]**

(b) Write down the vector equation for this object. **[No additional explanation needed.]**

(c) Write down a linear system of equations in the three variables x, y, and z that has this object as its solution set. In other words, if you solved the linear system that you write here, you would get the vector or parametric equations above as the solution. **[You need to show all your work and reasoning.]**