Math 214: Linear Systems **Midterm 3** Name: ______________________

Instructor: Ramin Naimi Friday, 16 Nov 2007

Closed book. Closed notes. NO CALCULATORS. Please write very legibly. You may use the back of each sheet for extra space.

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1. (25 points) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. Given that $\text{rref}(A) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$,
   (a) Find the dimensions of $\text{row}(A)$, $\text{col}\,(A)$, and $\text{null}(A)$. Explain your reasoning.
   (b) Is each of the following true or false? Just write T or F for each, without any explanations.
      i. $\text{row}(A) = \text{row}(\text{rref}(A))$
      ii. $\text{col}\,(A) = \text{col}\,(\text{rref}(A))$
      iii. $\text{null}(A) = \text{null}(\text{rref}(A))$
   (c) Is the vector $[3, 0, -3]$ in the row space of A? Explain your reasoning clearly.
   (d) Is the vector $\begin{bmatrix} 783 \\ -95 \end{bmatrix}$ in the column space of A? You may not do any computations! Explain your reasoning clearly.

2. (15 points) Show that $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$ and find the eigenvalue corresponding to this eigenvector.

3. (20 points) Prove that if a square matrix A is invertible then $\det(A) \neq 0$.

4. (15 points) Let T be the transformation given by $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \\ x+y \end{bmatrix}$.
   (a) Is T a linear transformation? Prove your answer.
   (b) Recall that $[T]$ represents the standard matrix of T. Which of the following is true? Just circle one without any explanations.
      i. $\dim(\text{range}(T)) > \dim(\text{col}\,([T]))$.
      ii. $\dim(\text{range}(T)) = \dim(\text{col}\,([T]))$.
      iii. $\dim(\text{range}(T)) < \dim(\text{col}\,([T]))$.
      iv. There isn't enough information to determine which of the above is true.

5. (25 points) Let $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.
   (a) Is $\{\vec{u}, \vec{v}\}$ an orthogonal set? Prove your answer.
   (b) Let $W = \text{span}(\vec{u}, \vec{v})$. Is $\{\vec{u}, \vec{v}\}$ a basis for W? Explain your reasoning very briefly.
   (c) Find a basis for $W^{\perp}$. Show all work. (Hint: We know two methods to do this: (i) Find a basis for the nullspace of some matrix. (ii) Take the cross product of some vectors. The second method is shorter. Whichever method you choose, briefly explain why the method gives a basis for $W^{\perp}$.)
   (d) Let $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Find the orthogonal decomposition of $\vec{b}$ with respect to W. Show and explain all work. (Hint: There two ways to do this: (i) Finding several projections and adding or subtracting some vectors. (ii) Finding just one projection and doing just one subtraction. The second way is shorter. You may choose either one.)