Test 1: Linear Systems

Math 214 Spring 2007 — Friday March 2

 — 2:30pm-3:25pm

Name: ________________________________

Directions: Read *all* problems first before answering any of them. There are 6 pages in this test. This is a 55-minute, no-notes, closed book, test. **No calculators.** You must show all relevant work to support your answers. Use complete English sentences and CLEARLY indicate your final answers to be graded from your "scratch work."

Pledge: I, ______________________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

No.	Score	Maximum
1		20
2		25
3		30
4		25
BONUS		10
Total		**100**

1. Span, Linear Independence, Rank. *20 points.*

Consider the matrix $A = \begin{bmatrix} 3 & 6 \\ 9 & 18 \end{bmatrix}$.

(a) (*4 points.*) Find the reduced row echelon form of A, $\text{rref}(A)$.

(b) (*4 points.*) With or without your knowledge of $\text{rref}(A)$, what is the rank of the matrix A? **EXPLAIN YOUR ANSWER.**

(c) (*4 points.*) With or without your knowledge of $\text{rref}(A)$, what is the span of the columns of matrix A? **EXPLAIN YOUR ANSWER.**

(d) (*4 points.*) With or without your knowledge of $\text{rref}(A)$, discuss the linear independence of the columns of the matrix A. **EXPLAIN YOUR ANSWER.**

(e) (*4 points.*) With or without your knowledge of $\text{rref}(A)$, discuss whether the matrix A^{-1} exists. **EXPLAIN YOUR ANSWER.**

2. Row reduction, Reduced Row Echelon Form, Identity, Invertibility. *25 points.*

(a) (*8 points.*) Show that $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & -1 \\ 2 & 2 & 6 \end{bmatrix}$ can be transformed into $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ using elementary row operations.

(b) (*8 points.*) Show that I_3 can be transformed into $B = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ using elementary row operations.

(c) (*9 points.*) Is it possible to transform A into B using elementary row operations? Explain why you can not or explain why you can, without going through the actual work of proving the result. Is A invertible? Is B invertible? How do you know? EXPLAIN YOUR ANSWERS.

3. Matrix Operations, Trace, Transpose. *30 points.*

TRUE or FALSE – put your answer in the box (1 point). To receive FULL credit, you must also give a brief, and correct, explanation in support of your answer! Remember if you think a statement is TRUE you must prove it is ALWAYS true. If you think a statement is FALSE then all you have to do is show there exists a counterexample which proves the statement is FALSE at least once.

The **trace** of a $n \times n$ matrix A (where A_{ij} is the element in the i^{th} row and j^{th} column) is defined as **the sum of the diagonal elements of A** and denoted $\text{tr}(A)$. In other words, $\text{tr}(A) = \sum_{i=1}^{n} A_{ii}$

(a) *10 points.* **TRUE or FALSE?** "The trace of the $n \times n$ identity matrix I_n, $\text{tr}(I_n)$ equals n."

$\boxed{\phantom{\text{TRUE or FALSE}}}$

(b) *10 points.* **TRUE or FALSE?** "$\text{tr}(A)\text{tr}(B) = \text{tr}(AB)$ for every $n \times n$ matrix A and B."

$\boxed{\phantom{\text{TRUE or FALSE}}}$

(d) *10 points.* **TRUE or FALSE?** "$\text{tr}(AA^T)$ equals the sum of the square of each element in A for every $n \times n$ matrix A."

$\boxed{\phantom{\text{TRUE or FALSE}}}$

4. Linear Systems, Equations of Planes and Lines. *23 points.*

Consider the linear system

$$\begin{aligned} 3x + 5y - 4z &= 0 \\ -3x - 2y + 4z &= 0 \\ 6x + y - 8z &= 0 \end{aligned}$$

(a) (*10 points.*) Find the non-trivial solution(s) of the linear system.

(b) (*10 points.*) What is the geometrical object which represents the solution of the linear system? Write down its equation in vector form.

(c) (*5 points.*) What is the smallest distance between the geometrical object which represents the solution of the linear system and the origin $(0, 0, 0)$. EXPLAIN YOUR ANSWER.

BONUS QUESTION. Analytic Geometry, Projections. (*10 points.*)
Find the smallest distance between the geometrical object which represents the solution to the linear system in Question 4 and the point $(1, 1, 1)$.