Math 214: Linear Systems **Midterm 1** Name: _____

Instructor: Ramin Naimi Friday, 21 Sept 2007

Closed book. Closed notes. NO CALCULATORS. Each problem is worth 25 points. You may also use the back of each sheet for writing your answers or for "scratch work" (but please indicate so clearly). Please write very legibly.

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1. (a) Are the vectors $[0, 1, 0, 0]$ and $[0, 0, 0, 1]$ orthogonal to each other in $\mathbb{R}^4$? Prove your answer.
   (b) Is the vector $[1, 1, 1, 1]$ a unit vector? If so, carefully explain why. If not, normalize it.

2. A plane P passes through the point $(1, 2, 3)$ and is perpendicular to the vector $[-1, 0, 1]$.
   (a) Write the normal form equation of P.
   (b) Find the general form equation of P. Show and explain your work.
   (c) Find the vector form equation of P. Show and explain your work.

3. Let P be the plane in $\mathbb{R}^3$ given by $3x + 2y + z = 4$.
   (a) Is the point $p = (1, 0, 1)$ on the plane P? Justify your answer.
   (b) Let $\vec{v}$ be the vector whose head is the point $q = (1, 0, 0)$ and whose tail is the point $p = (1, 0, 1)$. Find a vector $\vec{n}$ orthogonal to the plane P, and find $\text{proj}_{\vec{n}}(\vec{v})$. Show and explain all work.
   (c) Find the point on the plane P that is closest to the point $q = (1, 0, 0)$. Show and explain all work.

4. (a) Prove that if $\vec{v}$ and $\vec{w}$ are nonzero vectors in $\mathbb{R}^n$, then $\text{proj}_{\vec{v}}(\text{proj}_{\vec{v}}(\vec{w})) = \text{proj}_{\vec{v}}(\vec{w})$.
   (b) Explain geometrically why it makes sense that $\text{proj}_{\vec{v}}(\text{proj}_{\vec{v}}(\vec{w}))$ equals $\text{proj}_{\vec{v}}(\vec{w})$.