Class 21: Friday March 23

**TITLE** Introduction to Eigenvectors and Eigenvalues

**CURRENT READING** Poole 4.1

**Summary**
Let’s explore the wonderful world of *eigenvectors*, *eigenvalues* and *eigenspaces* of a square 2x2 matrix.

**Homework Assignment**

*HW #19 Poole, Section 4.1: 4, 5, 6, 10, 11, 16, 17, 21, 22. EXTRA CREDIT 36, 37*

**DEFINITION**
An **eigenvalue** of a \( n \times n \) matrix \( A \) is a scalar value \( \lambda \) such that there exists a non-zero vector \( \vec{x} \) where \( A\vec{x} = \lambda \vec{x} \). The vector \( \vec{x} \) is called the **eigenvector** corresponding to the **eigenvalue** \( \lambda \).

1. **Eigenvalues and Eigenvectors**

Interestingly, in order to find the eigenvalues of a matrix, one just has to solve the equation \( A\vec{x} - \lambda \vec{x} = \vec{0} \) or \( (A - \lambda I)\vec{x} = \vec{0} \).

This means that the eigenvectors of matrix \( A \) corresponding to eigenvalue \( \lambda \) lie in the nullspace of the matrix \( A - \lambda I \). It’s not clear right now, but it turns out that the eigenvalues of \( A \) are the solution of the equation \( \det(A - \lambda I) = 0 \). This equation is known as the **characteristic polynomial** of the matrix \( A \).

**EXAMPLE**

Find the eigenvalues and eigenvectors of \[
\begin{pmatrix}
1 & 2 \\
2 & 4
\end{pmatrix}
\]
2. Eigenvectors and Eigenspace

**Definition**
Given a $n \times n$ matrix $A$ with eigenvalue $\lambda$ the set of all vectors corresponding to the eigenvalue $\lambda$ plus the zero vector is called the **eigenspace** of $\lambda$ and is denoted $E_\lambda$.

**Exercise**
Write down the eigenspaces associated with the eigenvalues of $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

**Example**
Consider the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Let’s show that the eigenvalues of $A$ are the solution of $\lambda^2 - \text{tr}(A)\lambda + \det(A) = 0$.

3. Properties of the Eigenvalues of a Matrix

The **Product** of the eigenvalues equals the determinant of the matrix.

$$\lambda_1 \lambda_2 = |A|$$

The **Sum** of the eigenvalues equals the trace of the matrix (the sum of the diagonal entries)

$$\lambda_1 + \lambda_2 = \sum_{i=1}^{2} A_{ii}$$