Linear Systems

Math 214 Spring 2007 | Fowler 110 MWF 2:30pm - 3:25pm

 | http://faculty.oxy.edu/ron/math/214/07/

Class 20: **Wednesday March 21**

TITLE Application of Linear Algebra: Graph Theory
CURRENT READING Poole 3.6

Summary
Let's explore an interesting aplication of matrices to graph theory.

Homework Assignment
HW #18 Poole, Section 3.7 : 27, **28, 29**, *30, 35,* **36, 37**, *38.*

DEFINITION

A **graph** consists of a finite set of points called vertices and a finite set of edges which connect two (not necessarily distinct) vertices. Two vertices are said to be **adjacent** if they are the endpoints of an edge.

If G is a graph with n vertices, then its **adjacency matrix** is the $n \times n$ matrix A (sometimes written $A(G)$) defined by:

$$a_{ij} = \begin{cases} 1 & \text{if there is an edge between vertices } i \text{ and } j \\ 0 & \text{otherwise} \end{cases}$$

EXAMPLE

Consider the adjacency matrix $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$. Draw the associated graph.

DEFINITION

A **path** in a graph is defined to be a sequence of edges that allows travel from one vertex to another continuously. The **length** of a path is the number of edges the path contains. A path with k edges is called a **k-path**. When a path begins and ends with the same vertex it is called **closed** and the path is called a **circuit**. A path that does NOT include the same edge more than one is called a **simple path**.

Exercise

Show that $A^2 = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 2 & 3 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

Given A is the adjacency matrix of a graph G, the (i, j) entry in the matrix A^k is equal to the number of k-paths between vertices i and j.

EXAMPLE

Show that there are SIX 3-paths between the first vertex and second vertex.

DEFINITION

A graph with directed edges is known as a **digraph**. If G is a digraph with n vertices, then its **adjacency matrix** is the $n \times n$ matrix A (sometimes written $A(G)$) is defined by:

$$a_{ij} = \begin{cases} 1 & \text{if there is an edge FROM vertices } i \text{ TO } j \\ 0 & \text{otherwise} \end{cases}$$

Exercise

Given $A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ draw the corresponding directed graph below.

GROUPWORK

Consider the following 5-player **tournament** (a directed graph with exactly one directed edge between every pair of vertices) representing results between 5 tennis players Davenport, Graf, Hingis, Seles and (Venus) Williams. A directed edge from vertex i to vertex j means that Player i defeated Player j.

The reults are as follows:
Davenport defeated Graf, Seles, and Williams, lost to Hingis.
Graf defeated Hingis, Seles, and Williams, lost to Davenport.
Hingis defeated Davenport and Seles, lost to Graf and Williams.
Seles defeated Williams, lost to Davenport, Graf and Hingis.
Williams defeated Hingis, lost to Davenport, Graf and Seles.
(a) Draw a picture of the digraph representing these results.

(b) Write down the adjacency matrix for the digraph you drew. (What must all the entries of the main diaginal be?)

(c) Rank the players. **No Ties Allowed**. What matrix operations do you have to do comeup with a reasonable ranking order? DISCUSS YOUR STRATEGIES.