Linear Systems

Math 214 Spring 2007 Fowler 110 MWF 2:30pm - 3:25pm

http://faculty.oxy.edu/ron/math/214/07/

Class 9: **Monday February 12**

SUMMARY Applications of Linear Systems
CURRENT READING Poole 2.3 and 2.4

Summary
Now that we have formally defined linear independence and linear dependence and introduced the span concept, we can apply these concepts to linear systems in matrix form.

Homework Assignment
*HW #9: Section 2.4 4,***11,12***,31,32,39,***40,41***,46. EXTRA CREDIT 47. DUE WED FEB 14*

Recall that we ended *Class 8* by asking whether two given vectors are linear independent or not. Another way to answer this question is to use the result the text calls Theorem 2.6.

Theorem 2.6
Let $\vec{v_1}, \vec{v_2}, \ldots \vec{v_n}$ be column vectors in $\mathbb{R}^m$ and let the matrix A be the $m \times n$ matrix with these vectors as columns. The vectors $\vec{v_1}, \vec{v_2}, \ldots \vec{v_n}$ are **linearly dependent** IF AND ONLY IF the homogeneous linear system $A\vec{x} = \vec{0}$ with augmented matrix $[A|\vec{0}]$ has a *non-trivial solution* (i.e. one where $\vec{x} \neq \vec{0}$).

EXAMPLE
Determine whether $\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}$ are linearly independent or not.

Theorem 2.7
Let $\vec{v_1}, \vec{v_2}, \ldots \vec{v_m}$ be row vectors in $\mathbb{R}^n$ and let the matrix A be the $m \times n$ matrix with these vectors as rows. The vectors $\vec{v_1}, \vec{v_2}, \ldots \vec{v_n}$ are **linearly dependent** IF AND ONLY IF $\text{rank}(A) < m$.

Exercise
Use Theorem 2.7 to determine whether $\begin{bmatrix} 1 & 0 & 3 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} -1 & 1 & 3 \end{bmatrix}$ are linearly independent or not. (Look carefully. How are these vectors different from the ones in the EXAMPLE above?)

These results can be summarized in Theorem 2.8.

Theorem 2.8
Any set of m vectors in $\mathbb{R}^n$ is **linearly dependent** IF $m > n$. **Corollary.** It takes at least n vectors to span $\mathbb{R}^n$

GROUPWORK

Poole, Page 114, # 13. Balance the chemical reaction $Na_2CO_3 + C + N_2 \longrightarrow NaCN + CO$.

Poole, Page 114, # 39(a). Find the equation of the parabola which passes through the points $(0, 1)$, $(-1, 4)$ and $(2, 1)$.