Linear Systems

Class 1: **Monday January 22**

SUMMARY Scalars and Vectors
CURRENT READING Poole 1.1 and 1.2

INTRO
In today's class we review the concepts of vectors and scalars. In addition, we introduce the central idea of a **linear combination** of vectors.

Homework Assignment #1
Section 1.1 # 1d, 2d, 3c, 4c, 5a, 6, 9, **11, 15, 17, 20** *: DUE WED JAN 24*
EXTRA CREDIT #14

What is a vector?

Roughly speaking, a vector is just a "bunch of numbers"!
More precisely, a vector is an *ordered set of numbers*.

Example 1. $[2 \ 0]$ is a vector; $[0 \ 2]$ is also; these are two different vectors, since order matters.

$[2 \ -5 \ 7.1]$ is a **row vector**; $\begin{bmatrix} 2 \\ -5 \\ 7.1 \end{bmatrix}$ is a **column vector**.

Q: What's the difference between a row vector and a column vector?

Note. To save space, we sometimes write $(4, 0, -8)$ instead of $\begin{bmatrix} 4 \\ 0 \\ -8 \end{bmatrix}$. So $(4, 0, -8)$ is a column vector.

Each number in the vector is called a **component** of the vector.
Q: what's the second component of the vector $[3 \ 6 \ 0]$? **Ans:**

Vectors are used to represent many different things!

Example 2. Start from home. Drive 6 miles East, 2 miles North. Represent this by the vector $[6 \ 2]$. Then continue driving 3 miles East, 5 miles South. Represent this by $[3 \ -5]$.
Q: Where are we relative to home? **Ans:** Add the two vectors: $[6 \ 2] + [3 \ -5] = [9 \ -3]$.

(Draw picture)

• We add vectors component-wise: one component at a time.

Example 3. I have 4 nickels, 3 dimes, and 2 quarters. You give me 3 nickels and 1 dime, and take 1 quarter. So I'm left with: $[4 \ 3 \ 2] + [3 \ 1 \ -1] = [7 \ 4 \ 1]$.

Note. The book uses boldface letters for vectors. It is difficult to *write* in boldface. So instead we'll use "arrow notation" for vectors:
Book: Let $\mathbf{v} = [4 \ 3]$. Let $\mathbf{w} = [5 \ 3]$. Then $\mathbf{v} + \mathbf{w} = ?$
Us: Let $\vec{v} = [4 \ 3]$. $\vec{w} = [5 \ 3]$. Then $\vec{v} + \vec{w} = ?$

Example 4. $[4\ 2] + [3\ 1\ -1] =$? **Ans:** Undefined.

Vectors of different size can NOT be added to each other.

<center>Multiplying a vector by a number: scalars</center>

What's $5 + 5 + 5 + 5 + 5 + 5 =$?
What's $[5\ 3] + [5\ 3] + [5\ 3] + [5\ 3] + [5\ 3] + [5\ 3] =$?
So, what's $6[5\ 3] =$? **Ans:**

Here the number 6 is called a **scalar**. Why? Because if you draw both vectors, $[5\ 3]$ and $[30\ 18]$, on two separate xy-planes, they'll have different lengths but the same direction (slope): we're only changing the "scale on our map" to make one vector look like the other.

<center>Subtracting vectors</center>

Example 5. Let $\vec{v} = [4\ 3]$. $\vec{w} = [5\ 3]$. Then $\vec{v} - \vec{w} =$? Ans: $[-1\ 0]$.

How can we represent vector subtraction pictorially?

Step 1. Draw $\vec{v}$.
Step 2. Multiply $\vec{w}$ by -1.
Step 3. Add $-\vec{w}$ to $\vec{v}$.

<center>Linear Combinations</center>

Example 6. Find a and b such that $a[5\ 3] + b[3\ 2] = [0\ 1]$.

Ans: Solve two equations with two unknowns:
$5a + 3b = 0$
$3a + 2b = 1$.

We get: $a = -3$, $b = 5$.

So $(-3)[5\ 3] + (5)[3\ 2] = [0\ 1]$. We say $[0\ 1]$ is a *linear combination* of $[5\ 3]$ and $[3\ 2]$.
(Books sometimes just say combination, instead of linear combination.)

Definition 1. Let $\vec{v_1}, \cdots, \vec{v_n}$ be vectors. To say a vector $\vec{w}$ is a **linear combination** of $\vec{v_1}, \cdots, \vec{v_n}$ means there exist scalars $c_1, \cdots, c_n \in \mathbb{R}$ such that $c_1\vec{v_1} + \cdots c_n\vec{v_n} = \vec{w}$. The numbers $c_1, \cdots, c_n$ are called **coefficients**.

Example 7. Is $[5\ 6\ 0]$ a linear combination of $[1\ 0\ 0]$, $[0\ 3\ 0]$, and $[0\ 0\ 8]$? **Ans:**

Example 8. Is $[5\ 6\ 0]$ a linear combination of $[0\ 1\ 1]$, $[0\ 3\ 0]$, and $[0\ 0\ 8]$? **Ans:**

Example 9. What are all possible lin combs of $[1\ 0]$ and $[0\ 1]$? **Ans:**

Example 10. What are all possible lin combs of $[1\ 1]$ and $[2\ 2]$? **Ans:**